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Land Locomotion Laboratory
Research Division
Research and Engineering Directorate

ANALYSIS OF TOWED PNEUMATIC TIRES MOVING ON SOFT GROUND

by

M. G. Bekker

Z. Janosi

March 1960

Project No. 5510.11.822

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ACKNOWLEDGEMENT

Mr. Server Tasdemiroglu, Mr. James Landoll and Lt. Gordon Bassett helped with the tests and evaluation.

ABSTRACT

A theory by which pneumatic tire sinkage and motion resistance can be calculated has been developed. The formulae based on this theory have been verified by comparing experimental results obtained from tests conducted on several different soils with predicted results using the equations. The agreement between experimental and predicted results was acceptable.

PROJECT TITLE: ANALYSIS OF TOWED PNEUMATIC TIRES MOVING ON SOFT GROUND

I. BACKGROUND:

The observation that a pneumatic tire with a low inflation pressure performed better in soft soil than the same tire with a high inflation pressure is not new or uncommon. However, until recently the underlying relationships between the soil and the pneumatic tire had not been developed to allow adequate explanations of this phenomena or numerical predictions of sinkage, resistance, drawbar pull, etc. A study of the available literature reveals that numerous papers, studies, and test results have been published covering pneumatic tires. However, no method exists in this literature to predict the relationships between sinkage, motion resistance, inflation pressure and loading of a towed pneumatic tire in any given soil which would serve to explain the increase in performance associated with a decrease in inflation pressure. It was evident that although attempts have been made to develop a solution to obtain these definitions, the lack of a system or means to define soil strength hindered these efforts.

A purely empirical approach exists in an equation introduced by Omelianov (1). His equation would be suitable for prediction of motion resistance within restricted limits if the various parameters were known. The equation is:

$$R = c_1 W \sqrt[3]{\frac{p_i}{kD}} + c_2 W \sqrt[3]{\frac{W}{p_i D^2}}$$

where:

R is the resistance (Kg)

W is the load (Kg)

p_i is the inflation pressure (Kg/cm^2)

D is the diameter (cm)

k is a soil characteristic (Kg/cm^3)

c_1 and c_2 are dimensionless factors related to the tire.

The shortcomings of Omelianov's equation originate from the inadequacy of a single soil parameter in the mathematical description of cross-country mobility problems.

Because a theory and technique for performance evaluation of low inflated pneumatic tires is necessary, the Land Locomotion Laboratory has started a project to fill the gap. A discussion of this development and first results are presented in this paper.

II. THEORETICAL APPROACH:

A. Sinkage

The difference in the behavior of low and high inflation pressure tires is dependent upon the degree of tire deflection due to the load. A high inflation pressure relative to the ground pressure results in little deflection and the tire can be considered to operate as a rigid wheel. A tire with a low inflation pressure relative to ground pressure experiences large deflection and an extensive change in the character of the loading area. For this loading condition the tire must be treated as a track rather than a wheel. The relationship between the inflation pressure and the ground pressure must be considered in treating the pneumatic tire since a very definite difference in treatment exists between tires having low and high inflation pressure relative to ground pressure.

To determine sinkage of the pneumatic tire, we first consider the general relationship for the sinkage of a footing in soil. For this problem the ground contact area of a wheel shown in Figure 1 is taken as the footing.

The sum of the ground pressure under a footing is equal to the load on the footing.

$$W = \int_0^A p_g \, dA$$

where:

W is the load on the footing (lbs)

p_g is the ground pressure (psi)

A is the ground contact area (sq. in.)

The ground pressure may be expressed by a basic equation introduced by the Land Locomotion Laboratory, expressing an empirical soil stress-strain relationship for vertical displacement (2):

$$p_g = \left(\frac{k_c}{b} + k_\phi \right) z^n$$

where:

k_c is the cohesive modulus of sinkage $\left[\frac{\text{lbs}}{\text{in.}^{(n+1)}} \right]$

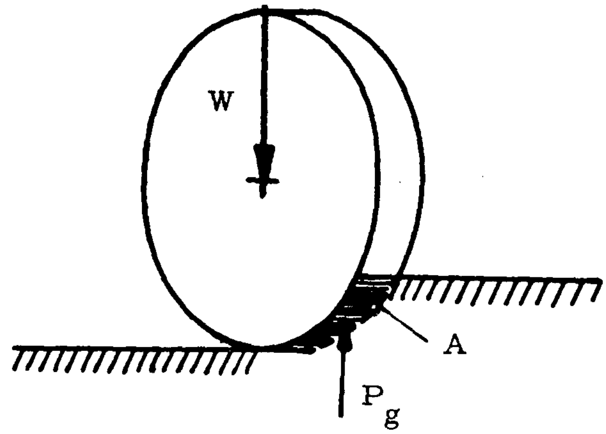

FIGURE 1. GROUND CONTACT AREA OF A WHEEL

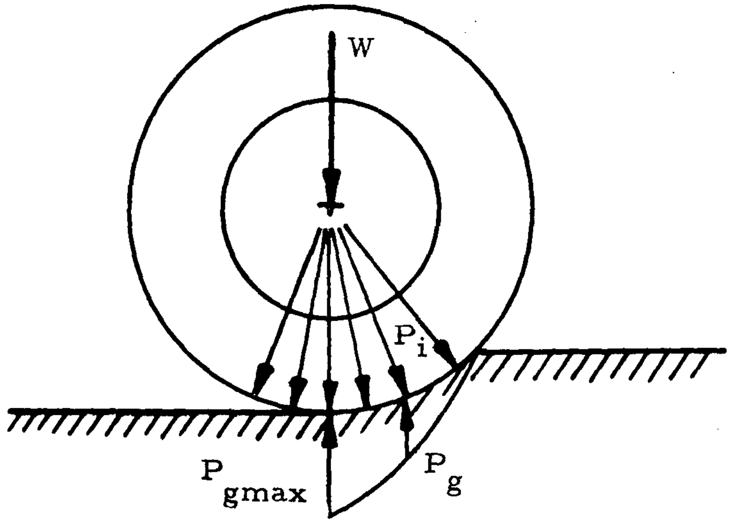

FIGURE 2. TIRE WITH HIGH INFLATION PRESSURE

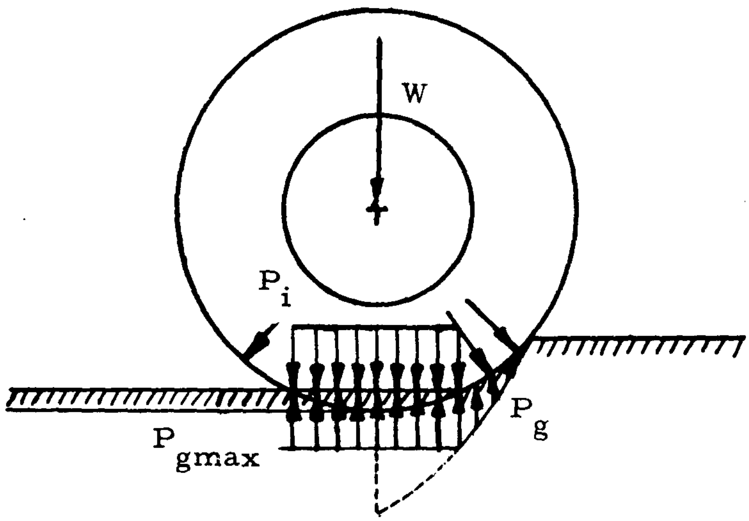

FIGURE 3. FLAT GROUND CONTACT AREA

k_ϕ is the frictional modulus of sinkage $\left[\frac{\text{lbs}}{\text{in.}^{(n+2)}}\right]$

b is the smaller size of the footing (in.)

z is the sinkage of the footing (in.)

n is the exponent of sinkage (dimensionless)

By combining equations 1 and 2 we obtain an expression relating load and sinkage:

$$W = \int_0^A p_g(z)dA = \int_0^A \left(\frac{k_c}{b} + k_\phi\right) z^n dA \qquad 3$$

It has been shown (3, 4) that in the case of a rigid wheel, equation 3 may be transformed to the following form:

$$W = z^{\left(\frac{2n+1}{2}\right)} \frac{(3-n)}{3} b \left(\frac{k_c}{b} + k_\phi\right) \sqrt{D}$$

or

$$z = \left[\frac{3W}{(3-n)\ (k_c + bk_\phi)\ \sqrt{D}}\right]^{\frac{2}{2n+1}} \qquad 4$$

where:

D is the diameter of the rigid wheel (in.)

Equation 4 may also be used for pneumatic tires of high inflation pressure. Such a tire is shown in Figure 2. It can be seen that when $p_i > p_{g\ max}$, the

tire has to be treated as a rigid wheel; where p_i is the inflation pressure, (psi).

However, if the load determines a sinkage (equation 4) that would result in $p_{g\ max}$ (equation 2) larger than p_i, the tire deflects and will have a flat ground contact area if the stiffness of the carcass is neglected (Figure 3).

The equilibrium of the ground contact area yields:

$$p_i = p_{g\ max}$$

But $p_{g\ max}$ is a function of the sinkage (equation 2).

$$p_i = p_{g\ max} = \left(\frac{k_c}{b} + k_\phi\right) z_o^{\,n}$$

and:

$$z_o = \left[\frac{p_i}{\frac{k_c}{b} + k_\phi}\right]^{\frac{1}{n}}$$

The test results show that the carcass pressure cannot be neglected without excessive inaccuracy. Therefore, the experimental value of the carcass pressure (p_c) has been introduced as done by other investigators (5).

The sinkage equation for a low inflated tire becomes:

$$z_o = \left[\frac{p_i + p_c}{\frac{k_c}{b} + k_\phi} \right]^{\frac{1}{n}} \qquad 5$$

J. S. Ageikin published a similar equation in the Russian magazine, Automotive Industry (6).

B. Resistance

The resistance to motion of a towed tire can be approximated under the following assumptions. The bulk of the resistance is caused by two effects: Compression of soil and the flexure of the soft tire. The compaction resistance (R_c) can be calculated using equation 3. The work (L) spent to compact the soil under the tire along ℓ distance can be obtained as follows:

$$L = \int_0^{\ell} \int_0^{A} p(z)dAd\ell$$

If z is constant along ℓ and the width of the rut (b) is also invariable

$$L = b\ell \int_0^{z_o} p(z)dz$$

and the compaction resistance:

$$R_c = \frac{L}{\ell} = b \int_0^{z_o} p(z)dz$$

Substituting equation 2:

$$R_c = b\left(\frac{k_c}{b} + k_\phi\right) \int_0^{z_o} z^n dz$$

This yields:

$$R_c = \frac{(k_c + bk_\phi)}{n+1} z_o^{\,n+1} \qquad 6$$

If equation 5 is substituted into equation 6 the sinkage (z) can be eliminated:

$$R_c = \frac{\left[b(p_i + p_c)\right]^{\frac{n+1}{n}}}{(n+1)(k_c + bk_\phi)^{1/n}} \qquad 7$$

The deflection resistance (R_d) is due to carcass deflection and can be only obtained experimentally. It has been found that the following equation is accurate enough for practical purposes:

$$R_d = W \frac{\mu}{(p_i)^a} \qquad 8$$

where μ and a are experimental factors.

The resistance can now be approximated as:

$$R = R_c + R_d = \frac{\left[b(p_i + p_c)\right]^{\frac{n+1}{n}}}{(n+1)(k_c + bk_\phi)^{\frac{1}{n}}} + W \frac{\mu}{(p_i)^a} \qquad 9$$

C. Optimum Inflation Pressure

Since the resistance is a function of the inflation pressure, an optimum inflation pressure (p_{io}) can be found at which the resistance is minimal. Differentiating equation 9 with respect to pressure we have:

$$\frac{dR}{dp_i} = \frac{b\left[\frac{n+1}{n}\right]\left[b(p_i + p_c)\right]^{\frac{1}{n}}}{(n+1)(k_c + bk_\phi)^{\frac{1}{n}}} - W \frac{a\mu}{p_i^{a+1}}$$

when:

$$\frac{dR}{dp_i} = 0$$

$$(p_{io} + p_c)(p_{io})^{n(a+1)} = \left[\frac{nWa\mu}{b}\right]^n \left[\frac{k_c + bk_\phi}{b}\right] \qquad 10$$

From which p_{io} can be found if the other values are known.

D. Critical Pressure

As stated previously, a tire is treated as a rigid wheel or as a soft tire depending on the inflation pressure if other conditions are fixed. Therefore,

a critical pressure must exist that separates the two cases. The following considerations lead to an equation that gives the approximate order of this limiting pressure.

The mechanical equilibrium of the tire (Figure 4) can be expressed by three equations:

$$W \quad b\ell_1(p_i + p_c) + b \int_o^{\ell_2} p_x dx \qquad 11$$

$$H = R = \frac{\left[b(p_i + p_c)\right]^{\frac{n+1}{n}}}{(n+1)\ (k_c + bk_\phi)^{\frac{1}{n}}} + W \frac{\mu}{(p_i)^a} \qquad 12$$

$$Hd - Wa + b(p_i + p_c)\frac{\ell_1^{\,2}}{2} - b\int_o^{\ell_2} p_x x dx - Rc = 0 \qquad 13$$

Some simplifications are now introduced to obtain a solution for equations 11, 12, and 13:

$$c = \frac{z_o}{\beta}\ ; \quad a = \frac{1}{2}\ ; \quad \frac{z_o - z_x}{x} = \frac{z_o}{\ell_2}$$

where β depends on n and equals the distance between the center of gravity of the p = f (z) curve and the bottom of the root (Figure 5). If n = 1 and β = 3.

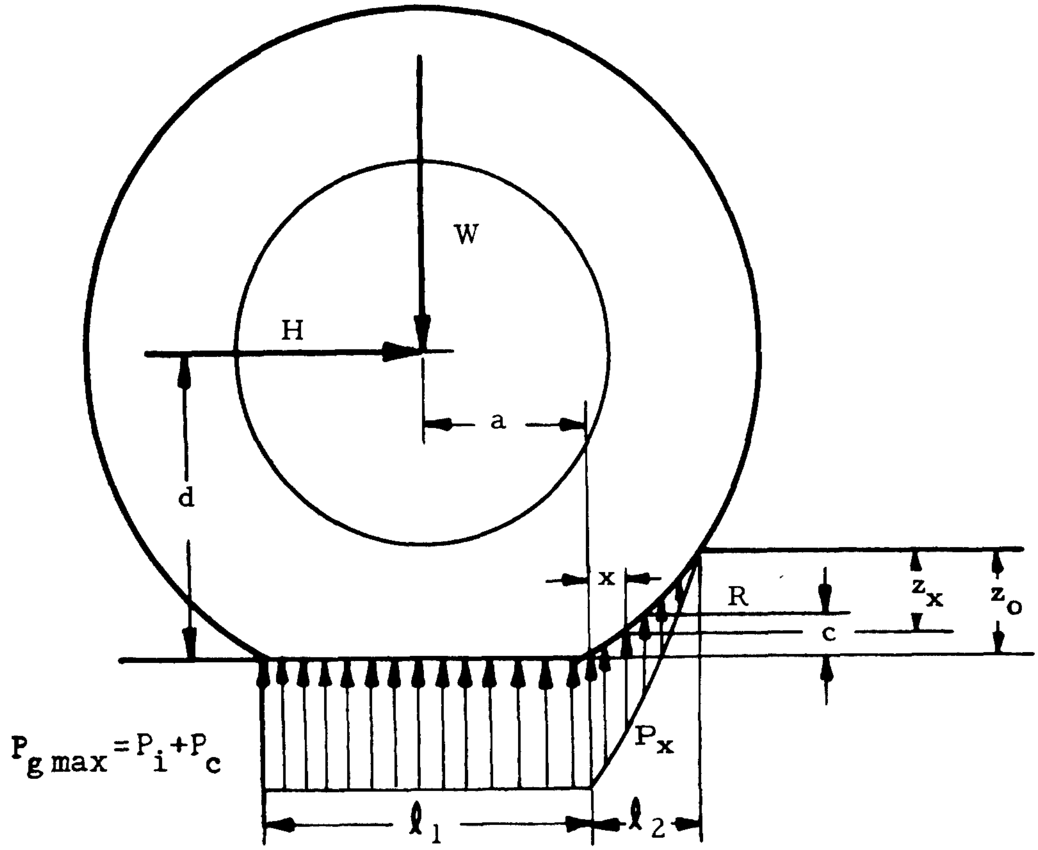

FIGURE 4. MECHANICAL EQUILIBRIUM OF A TOWED TIRE

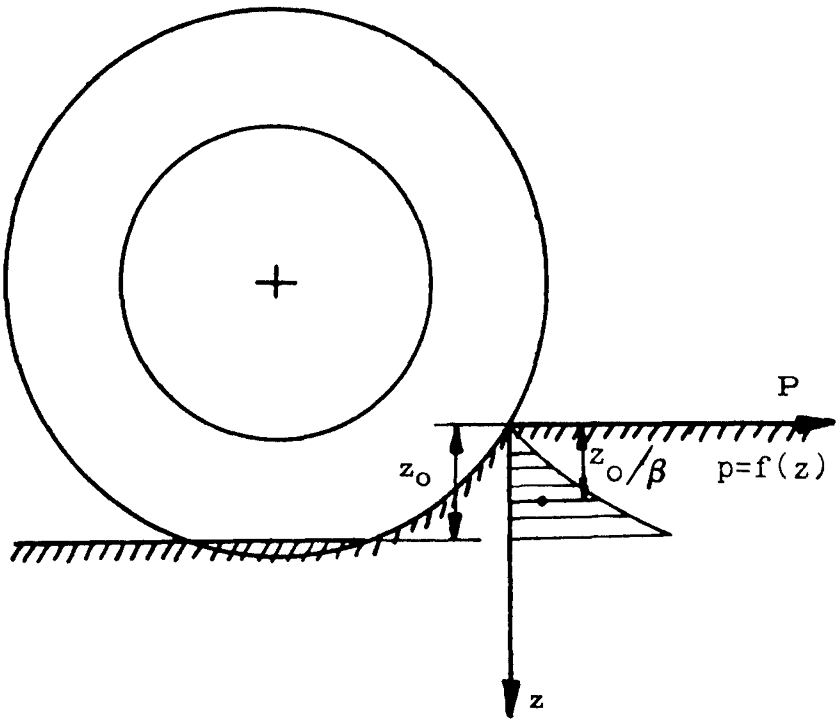

FIGURE 5. PRESSURE DISTRIBUTION UNDER THE TIRE

The integral involved in equation 11 can be solved as follows:

$$b\int_0^{\ell_2} p_x dx = b\left(\frac{k_c}{b} + k_\phi\right)\int_0^{\ell_2} z_x^n dx = b\left(\frac{k_c}{b} + k_\phi\right) z_o^n \int_0^{\ell_2} \left(1-\frac{x}{\ell_2}\right)^n dx$$

but:

$$b\left(\frac{k_c}{b} + k_\phi\right) z_o^n = b(p_i + p_c)$$

denoting:

$$y = \left(1-\frac{x}{\ell_2}\right) \text{ and, } dx = -\ell_2 dy$$

and the integration leads to:

$$\frac{b(p_i + p_c)\,\ell_2}{(n+1)} \qquad 14$$

The solution of the integral in equation 13 can be obtained in a similar way, and yields the following expression:

$$-\frac{b(p_i + p_c)\ell_2^2}{(n+1)(n+2)}$$

Equation 11 becomes:

$$W = b(p_i + p_c)\left(\ell_1 + \frac{\ell_2}{n+1}\right)$$

Using equation 12 and the result of the second integral, another equation can be obtained from equation 13:

$$\left\{ \frac{\left[b(p_i + p_c)\right]^{\frac{n+1}{n}}}{(n+1)(k_c + bk_\phi)^{\frac{1}{n}}} + W \frac{\mu}{p_i^{\,a}} \right\} \left[\frac{D}{2} - \frac{1}{3} \left(\frac{p_i + p_c}{\frac{k_c}{b} + k_\phi} \right)^{\frac{1}{n}} \right]$$

$$+ W \frac{\ell_1}{2} + b(p_i + p_c) \left[\frac{\ell_1^2}{2} - \frac{\ell_2^2}{(n+1)(n+2)} \right] = 0 \qquad 15$$

Using the following notation:

$$J = \frac{D}{2} - \frac{1}{3} \left(\frac{p_i + p_c}{\frac{k_c}{b} + k_\phi} \right)^{\frac{1}{n}} ; \quad F = \frac{1}{n + 1} ;$$

$$K = \frac{1}{(n + 1)(n + 2)}$$

and substituting ℓ_1 from equation 14 into equation 15:

$$IJ - \frac{W}{2} \left[\frac{W}{b(p_i + p_c)} - F\ell_2 \right] + b \left(p_i + p_c \right)$$

$$\left[\frac{1}{2} \left(\frac{W}{b(p_i + p_c)} - F\ell_2 \right)^2 - K\ell_2^{\,2} \right] = 0 \qquad 16$$

$$RJ - \ell_2 \frac{WF}{2} + \ell_2^2 \left[\frac{F}{2} - K\right] b(p_i + p_c) = 0 \qquad 17$$

Equation 17 is an equation of second degree in ℓ_2 so:

$$\ell_2 = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \qquad 18A$$

where:

$$B = \frac{-WF}{2} \; ; \; A = \left[\frac{F^2}{2} - K\right] b(p_i + p_c) \; ; \; C = RJ$$

The positive value of the square root yields a negative solution, which has no physical meaning.

Solving equation 14 for ℓ_1 :

$$\ell_1 = \frac{W}{b(p_i + p_c)} - F\ell_2 \qquad 18B$$

Equations 18A and 18B could be used for drawbar pull calculations since the ground contact area is now determined. Equation 18B can also serve as a basis for computing the deflection. From the triangle in Figure 6.

$$\delta = \frac{D}{2} - \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{\ell_1}{2}\right)^2} = \frac{1}{2} \left(D - \sqrt{D^2 - \ell_1^2}\right) \qquad 19$$

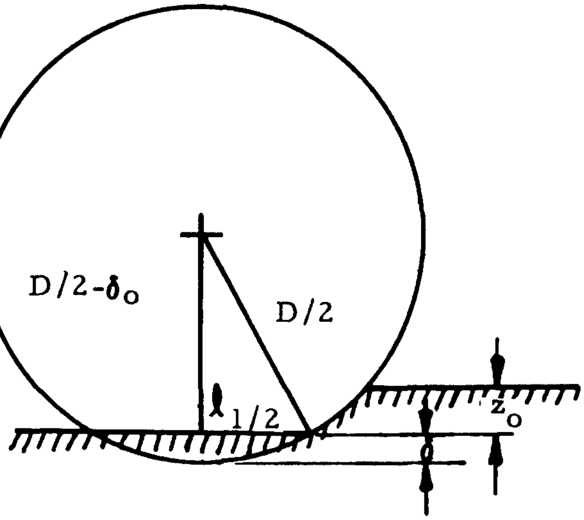

FIGURE 6. GEOMETRICAL CONFIGURATION OF A DEFLECTED TIRE

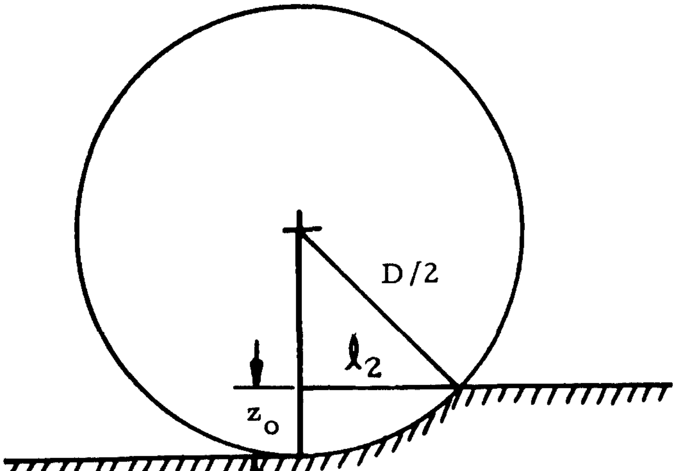

FIGURE 7. GEOMETRICAL CONFIGURATION OF A UNDEFLECTED TIRE

Equation 18B also shows that with higher inflation pressure (p_i) the length ℓ_1 of the flat portion of the ground contact area decreases, for:

$$p_i = p_{critical}; \ \ell_1 = 0$$

and:

$$\frac{W}{b(p_i+p_c)} = F\ell_2 \qquad 20$$

When the deflection is zero, no carcass pressure occurs ($p_c = 0$) and ℓ_2 can be found as shown in Figure 7:

$$\ell_2 = \sqrt{Dz_o - z_o^2} \qquad 21$$

Equations 20 and 21 now yield:

$$p_{crit} = \frac{W}{Fb\sqrt{Dz_o - z_o^2}} \qquad 22$$

Since $F = \frac{1}{n+1}$, and z can be substituted from equation 4, our final equation reads as follows:

$$p_{crit} = \frac{(n+1)W}{b\left(D\left[\frac{3W}{(3-n)(k_c+bk_\phi)\sqrt{D}}\right]^{2/2n+1} - \left[\frac{3W}{(3-n)(k_c+bk_\phi)\sqrt{D}}\right]^{4/2n+1}\right)^{1/2}}$$

III. TESTS.

In order to check the validity of equations 5 and 8, numerous experiments were performed. The test tire was tested in three different types of soil which were classified by the sinkage parameters of the Land Locomotion Soil-Value System; k_c, k_ϕ and n (2). The first soil was sand characterized by $k_c = 0$, $k_\phi = 7.0$ and $n = 0.8$. The second soil was an artificial soil (7) and its soil values were $k_c = 21\ 0$, $k_\phi = 4.0$ and $n = 0.43$. Finally, natural soil was used that had the following soil values: $k_c = 9.9$, $k_\phi = 13.2$, and $n = 0.86$

A 7.00 x 16 tire with the tread stripped off to eliminate its haphazard effects, was used. The tests were conducted in a 10 x 3 x 1 ft. soil bin. A simple carriage (Figure 8) to permit variation in wheel load was constructed and used throughout the tests The carriage was pulled by means of a spring scale as shown in Figure 9. Each test was performed in the following manner:

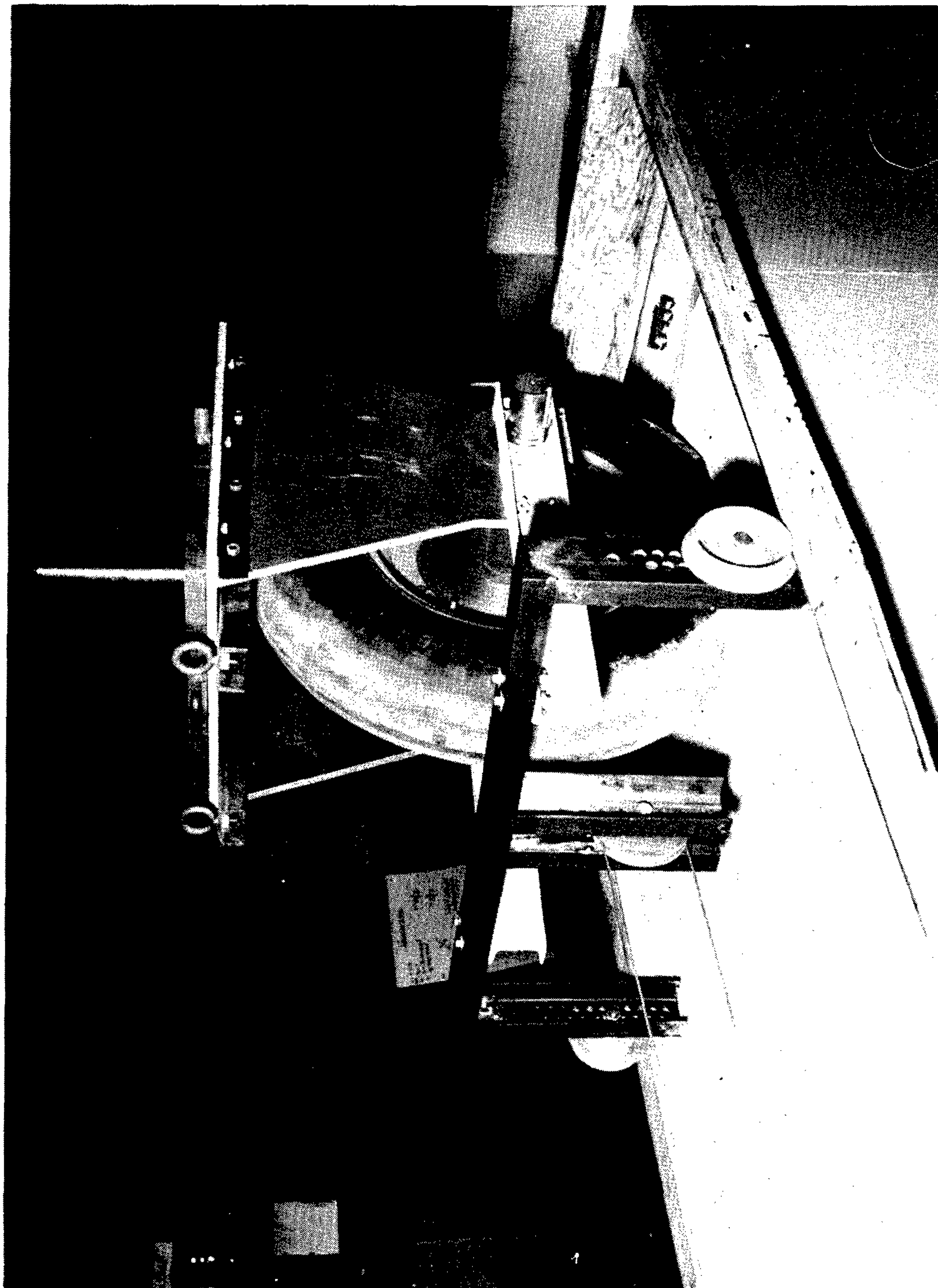

FIGURE 8. TEST CARRIAGE

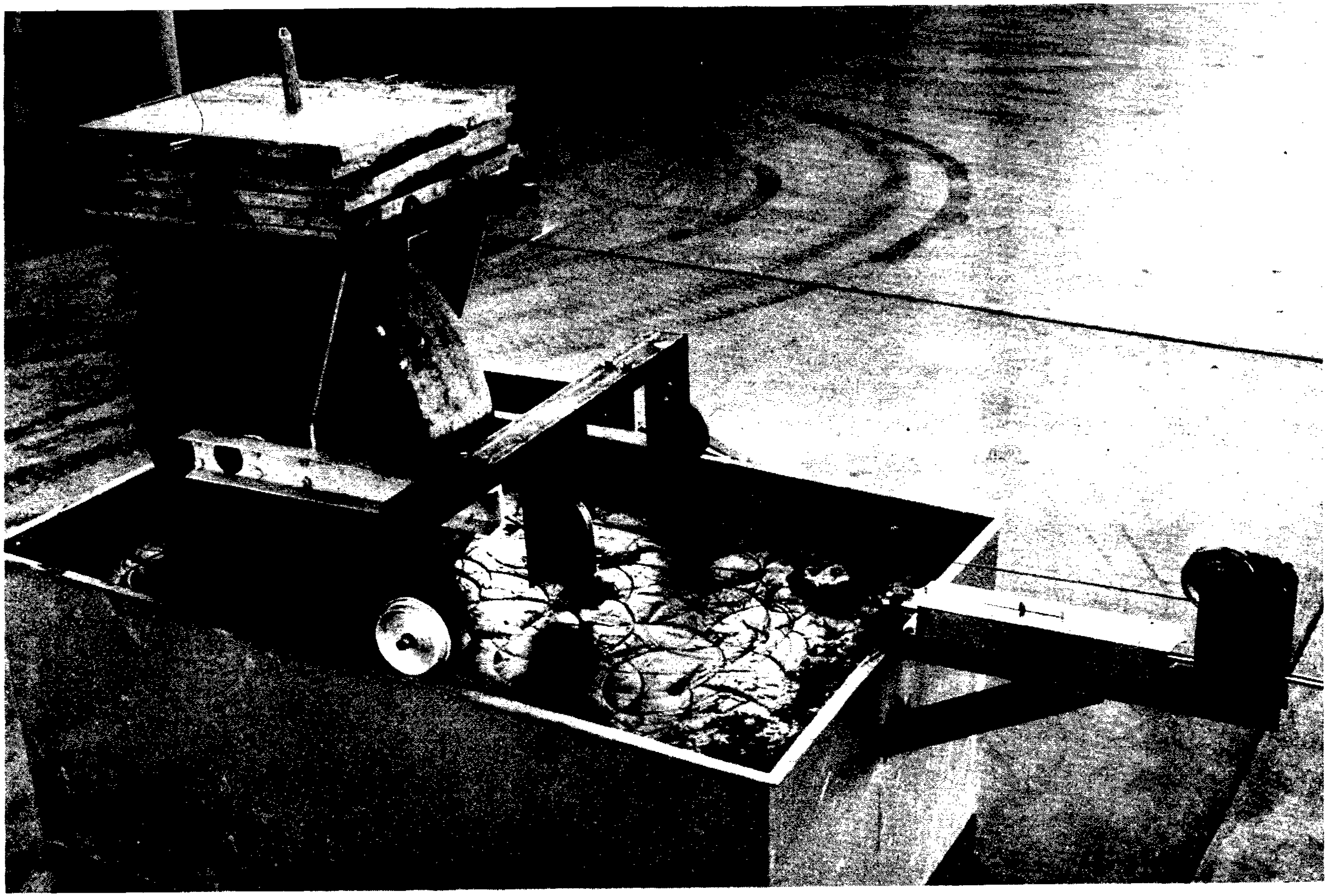

FIGURE 9. TEST CARRIAGE AND SPRING SCALE

A. The load on the tire was measured while the front bars of the carriage were exactly horizontal.

B. The dimensions of the print area were measured on hard ground. Fine sand was poured around the ground contact area to identify the area so that when the tire was lifted the print-area was easy to measure (Figure 10).

C. Next, the tire deflection (δ) was obtained on hard ground.

D. A large steel plate was placed on the soil horizontally to approximate hard ground conditions. The carriage was set on the plate and the front bars were adjusted to horizontal position. Then, the carriage was towed on the steel plate and the motion resistance due to tire deflection recorded.

E. The plate and the carriage were removed, and the soil was loosened and leveled. The height of the soil level was marked relative to a reference point and the carriage was placed on the soil. The front

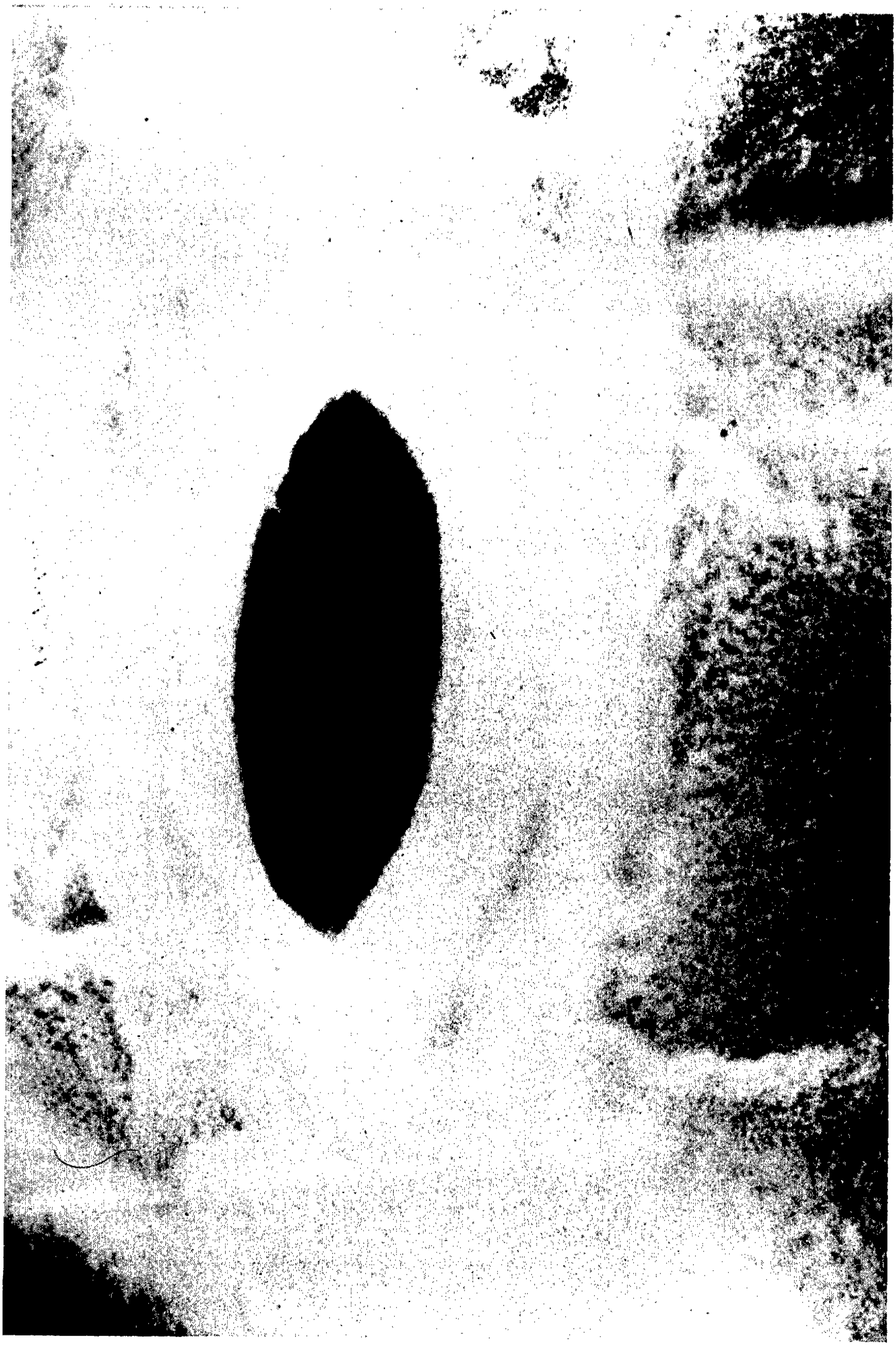

FIGURE 10. TIRE PRINT

bars were again adjusted, and the carriage was pulled along the bin (Figure 10). At the end of the run the carriage was immediately lifted. The total motion resistance was determined by the spring scale and the distance was measured between the bottom of the root and the reference point mentioned previously. Sinkage was determined from the difference between the soil level to reference point and root to reference point distances. Figure 11 shows a tire moving in sand.

F. Finally, the number of weights and/or the inflation pressure was changed for the next run and the procedure repeated.

IV. DISCUSSION OF TEST RESULTS:

Although several simplifying assumptions were made for equation 5, adequate agreement between test results and theoretical predictions was found. Experiments and predicted values are plotted in Figures 12 through 21. Equation 5 yielded somewhat higher sinkage values than actual tests in sand,

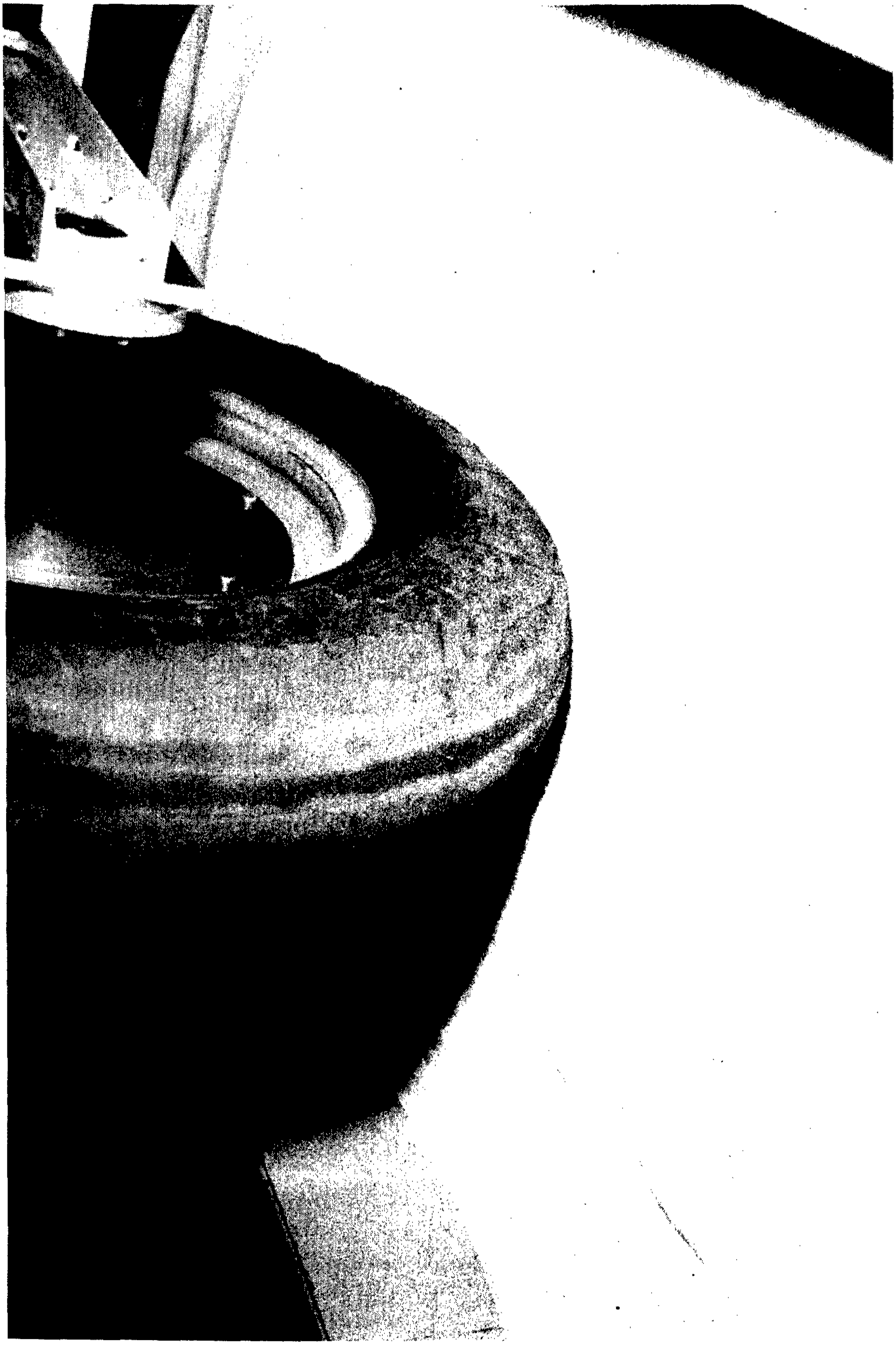

FIGURE 11. SOFT TIRE IN SAND

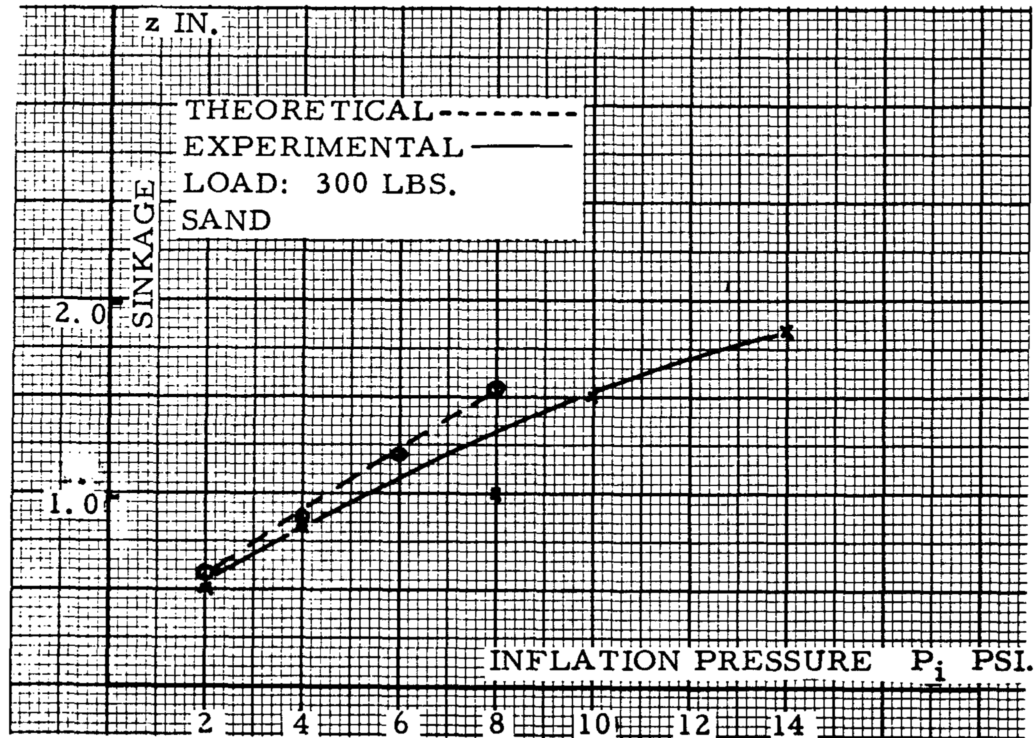

FIGURE 12. EXPERIMENTAL AND THEORETICAL RESULTS, 300-LB. LOAD, SAND

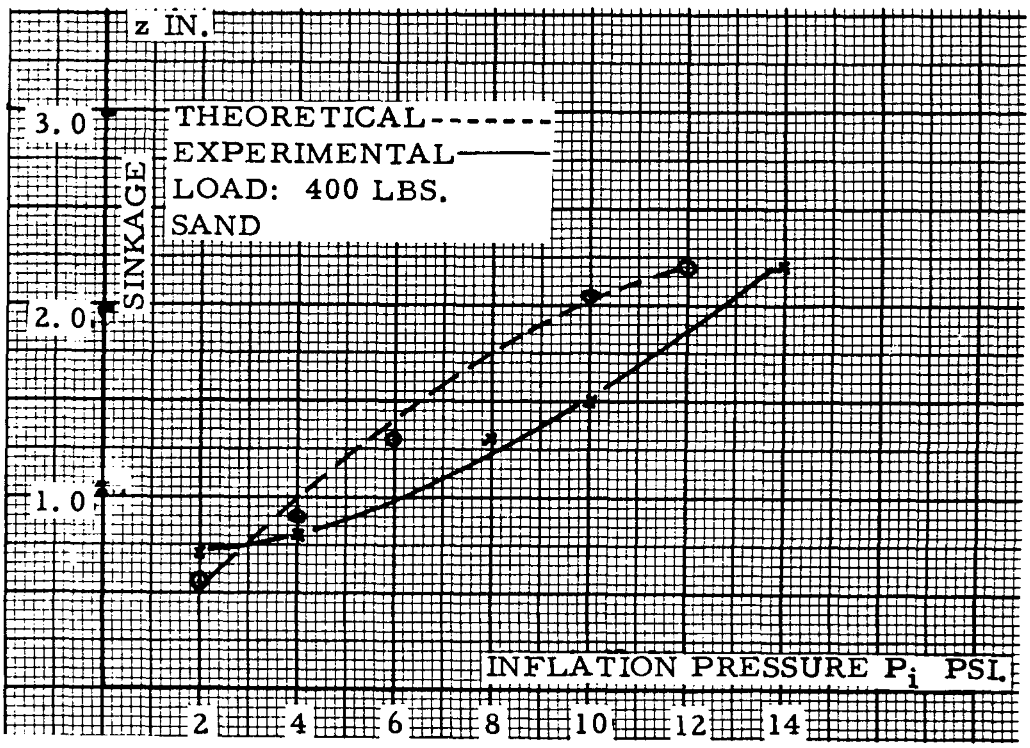

FIGURE 13. EXPERIMENTAL AND THEORETICAL RESULTS, 400-LB. LOAD, SAND

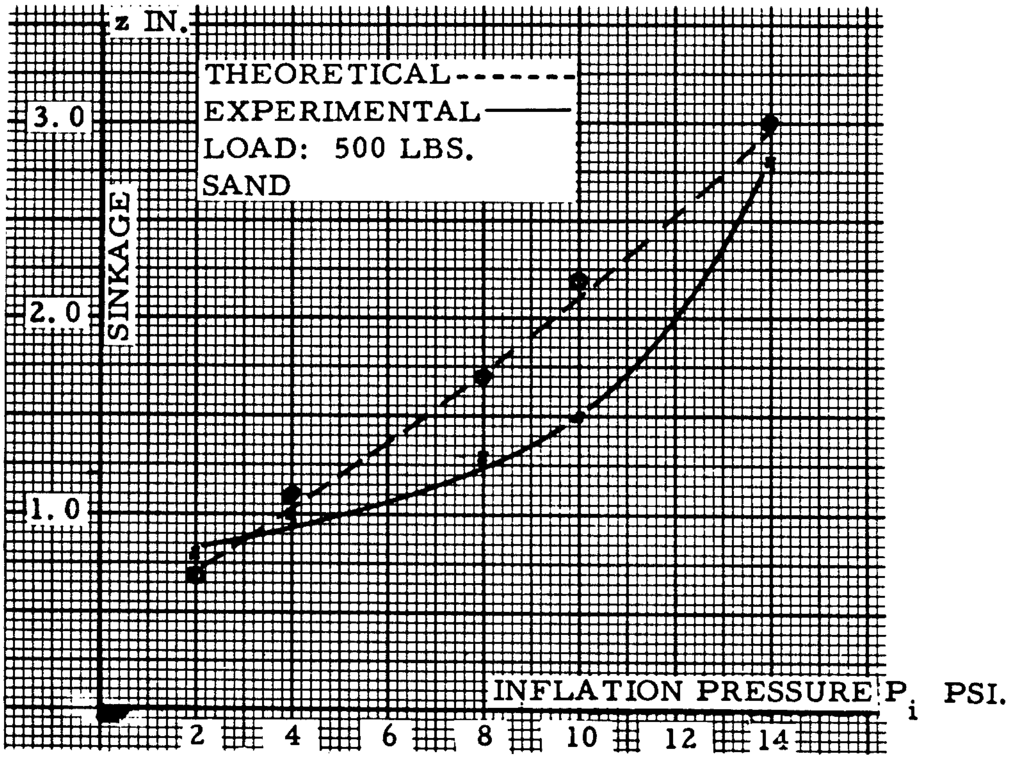

FIGURE 14. EXPERIMENTAL AND THEORETICAL RESULTS, 500-LB. LOAD, SAND

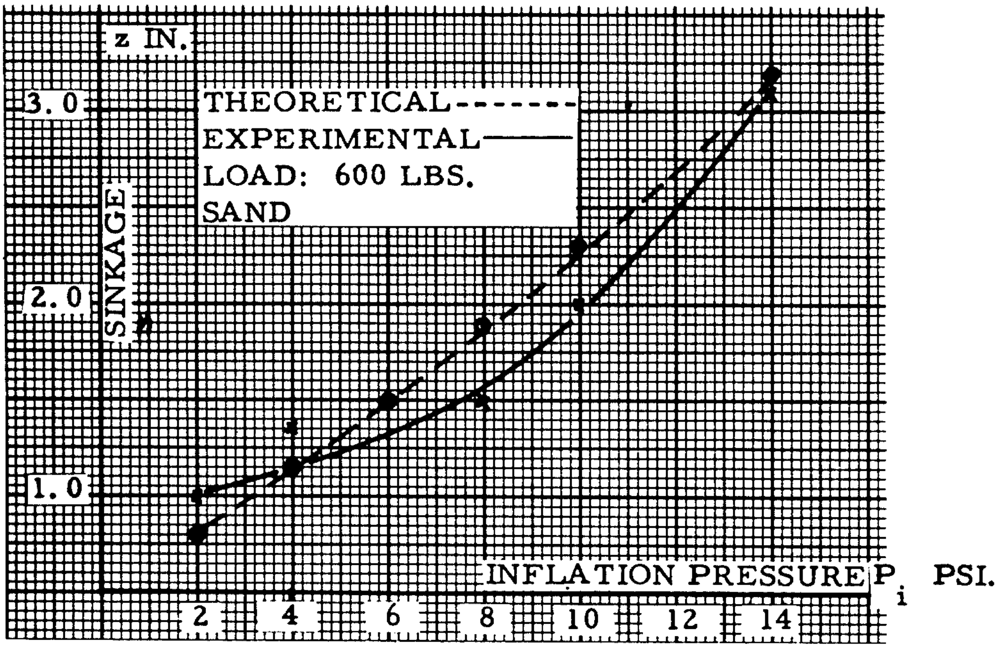

FIGURE 15. EXPERIMENTAL AND THEORETICAL RESULTS, 600-LB. LOAD, SAND

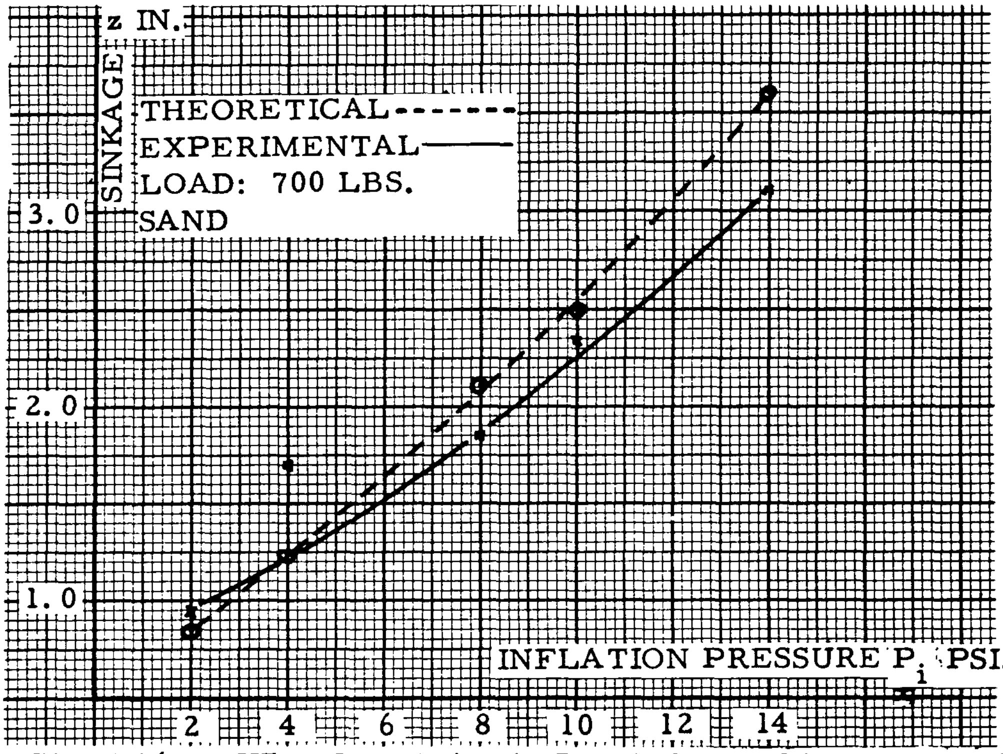

FIGURE 16. EXPERIMENTAL AND THEORETICAL RESULTS, 700-LB. LOAD, SAND

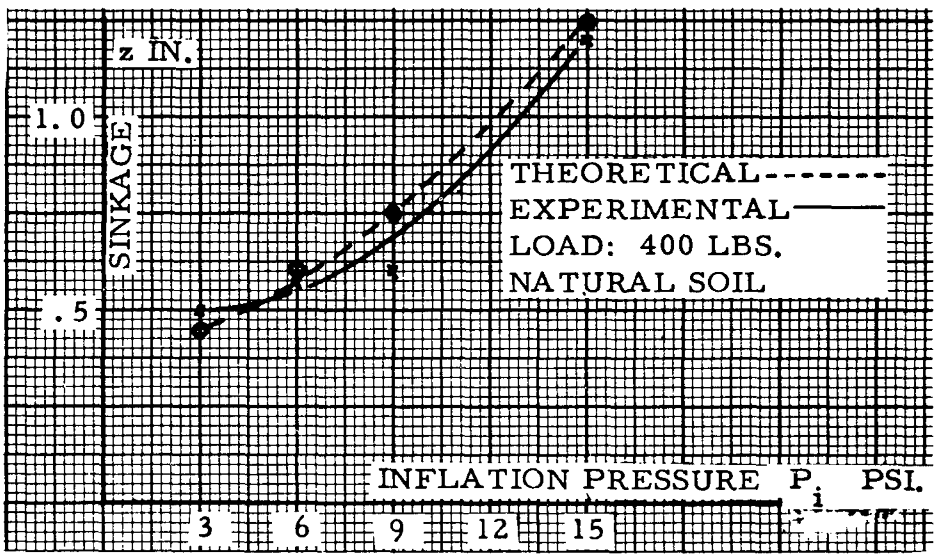

FIGURE 17. EXPERIMENTAL AND THEORETICAL RESULTS, 400-LB. LOAD, NATURAL SOIL

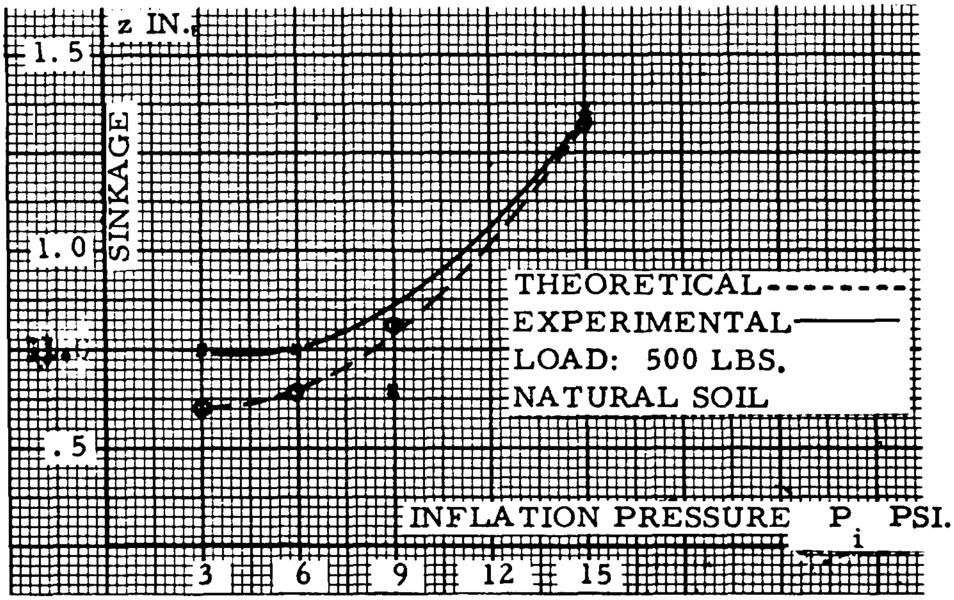

FIGURE 18. EXPERIMENTAL AND THEORETICAL RESULTS, 500-LB. LOAD, NATURAL SOIL

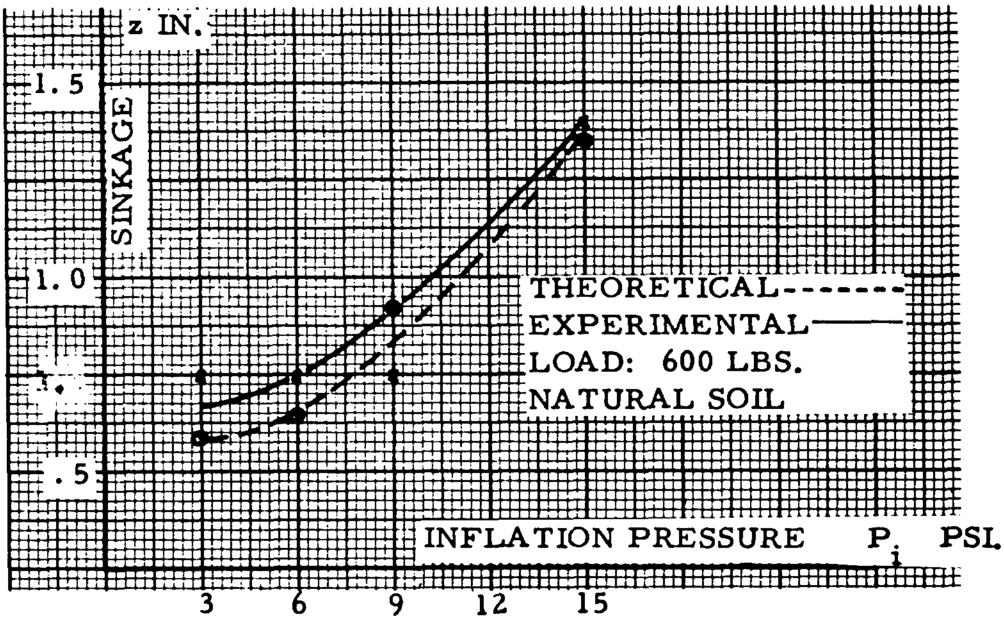

FIGURE 19. EXPERIMENTAL AND THEORETICAL RESULTS, 600-LB. LOAD, NATURAL SOIL

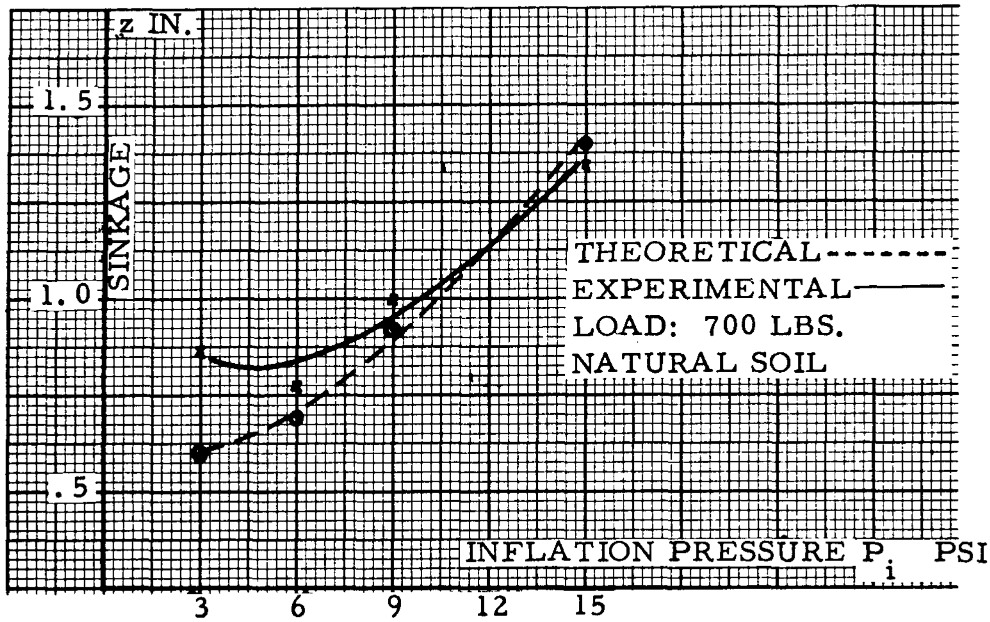

FIGURE 20. EXPERIMENTAL AND THEORETICAL RESULTS, 700-LB. LOAD, NATURAL SOIL

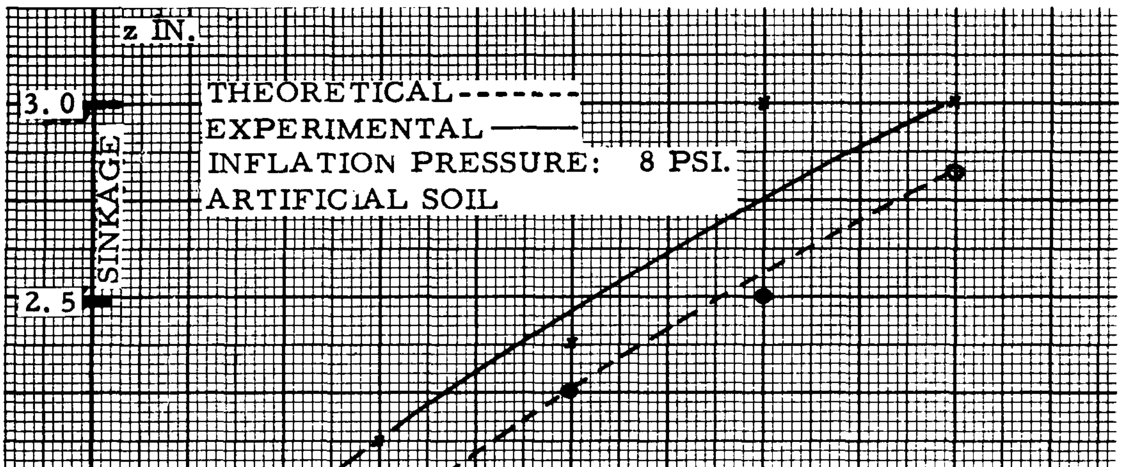

FIGURE 21. EXPERIMENTAL AND THEORETICAL RESULTS, WEIGHT VS SINKAGE ARTIFICIAL SOIL

particularly at lower loads and higher inflation pressures. The agreement is quite satisfactory for higher loads. Predictions were well justified for natural and artificial soils.

Figures 22 through 26 are plots of tire print area on a rigid surface as a function of the inflation pressure for various loads. The carcass pressure is the difference between the average ground pressure $\frac{W}{A}$ and the inflation pressure:

$$p_c = \frac{W}{A} - p_i$$

As can be seen:

$$p_c = \frac{W}{A} - p_i = \text{constant}$$

for a given load, so we may consider the carcass pressure as independent of inflation pressure. Consequently, the ground pressure $(p_i + p_c)$ can be assumed unchanged for hard and soft ground operation. Therefore, the use of a p_c value obtained from hard ground test seems to

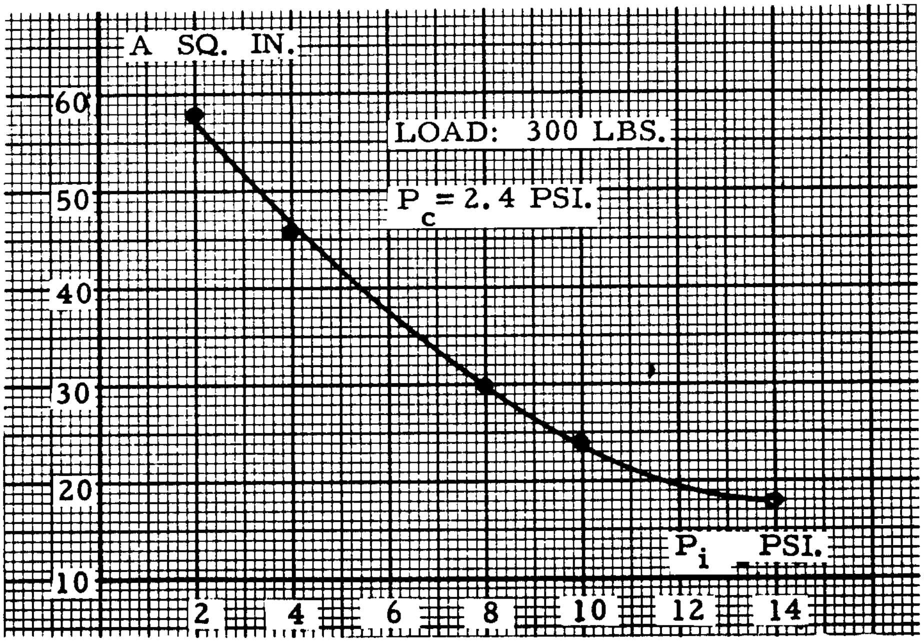

FIGURE 22. TIRE PRINT AREA ON RIGID SURFACE, 300-LB. LOAD

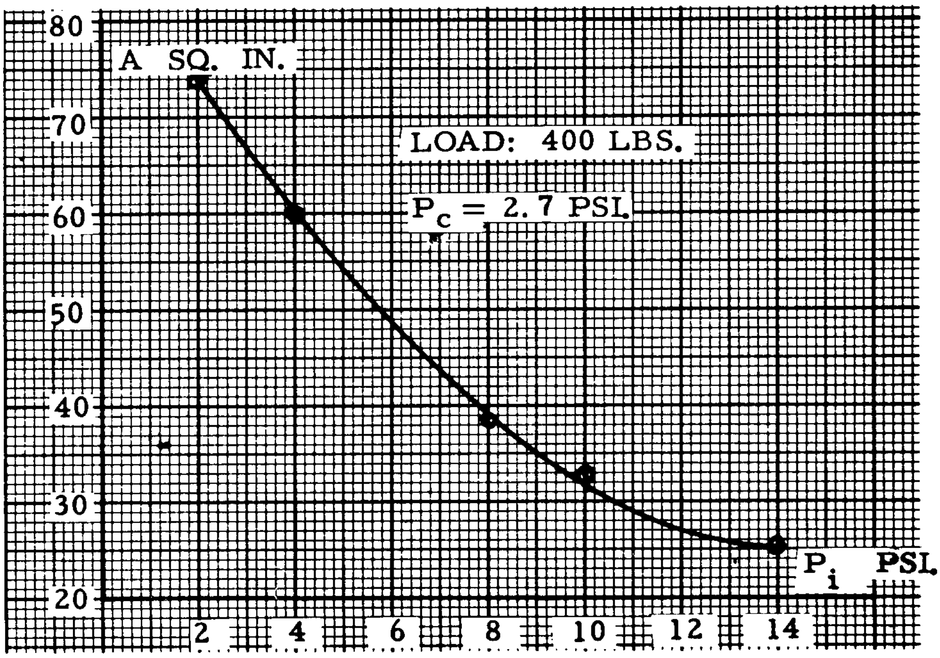

FIGURE 23. TIRE PRINT AREA ON RIGID SURFACE, 400-LB. LOAD

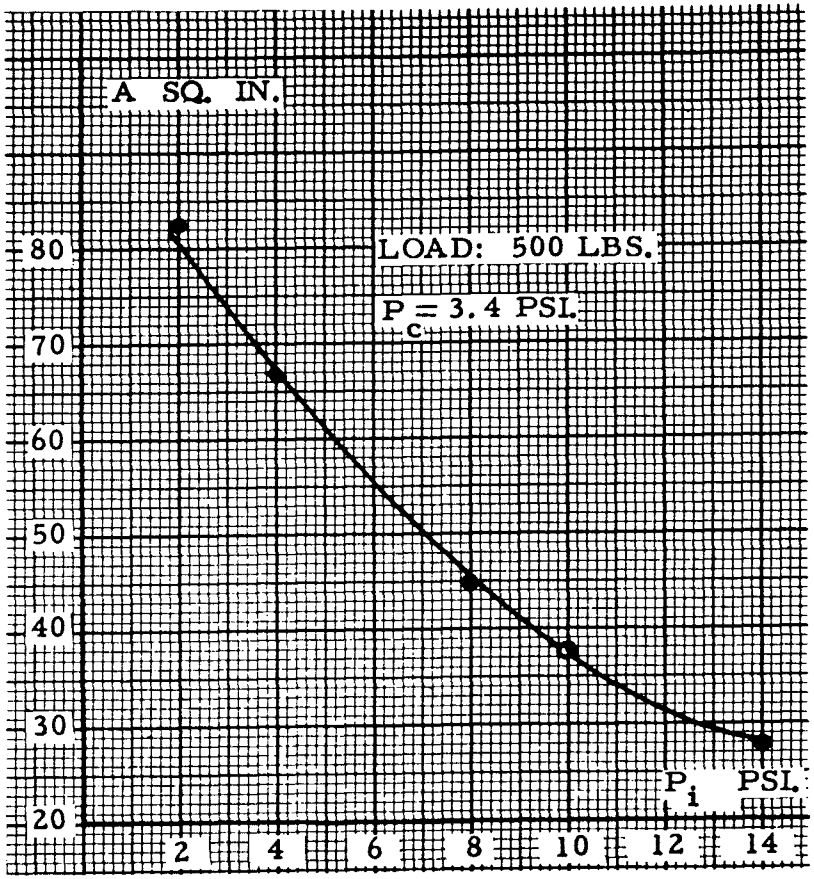

FIGURE 24. TIRE PRINT AREA ON RIGID SURFACE, 500-LB. LOAD

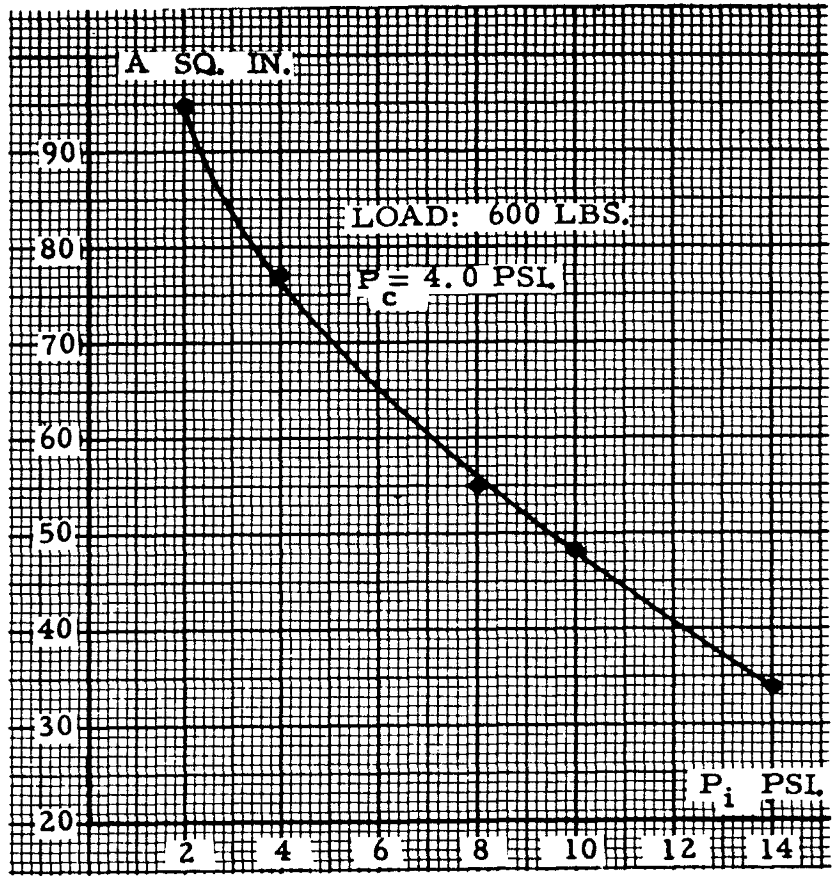

FIGURE 25. TIRE PRINT AREA ON RIGID SURFACE, 600-LB. LOAD

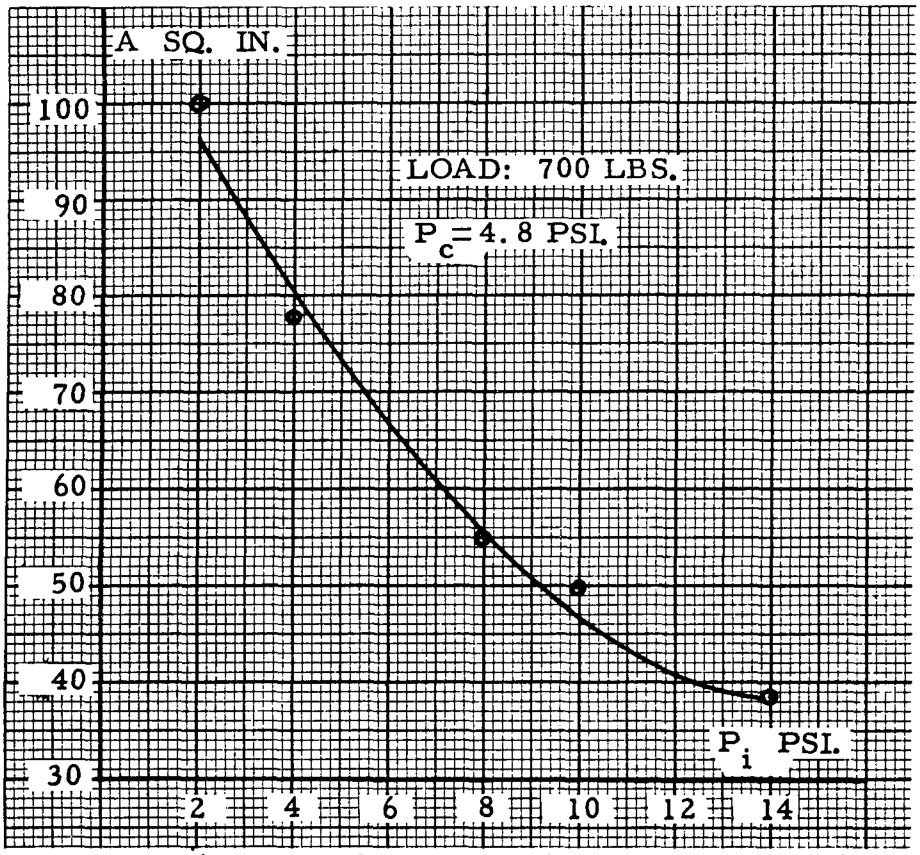

FIGURE 26. TIRE PRINT AREA ON RIGID SURFACE, 700-LB. LOAD

be justified for all sinkage predictions. The main reason for the disagreement between theoretical and experimental values for tests conducted in sand is that the critical pressure defined by equation 23 is 12.3 psi for W = 300 lbs in sand. Theoretical predictions could not have been in agreement with tests when the inflation pressure was higher than 12 psi. Furthermore, at an inflation pressure of 9.6 psi the addition of the carcass pressure (2.4 psi in this case) yields about 12 psi ground pressure and an almost rigid tire. The application of the rigid wheel equation (equation 4) seems to be proper for inflation pressures higher than $p_{cr} - p_c$.

The results of resistance predictions (equation 9) and actual tests are compared in Figures 27 through 37. As in the case of sinkage, the agreement between predicted and experimental results is more satisfactory for the natural and artificial soils than for sand.

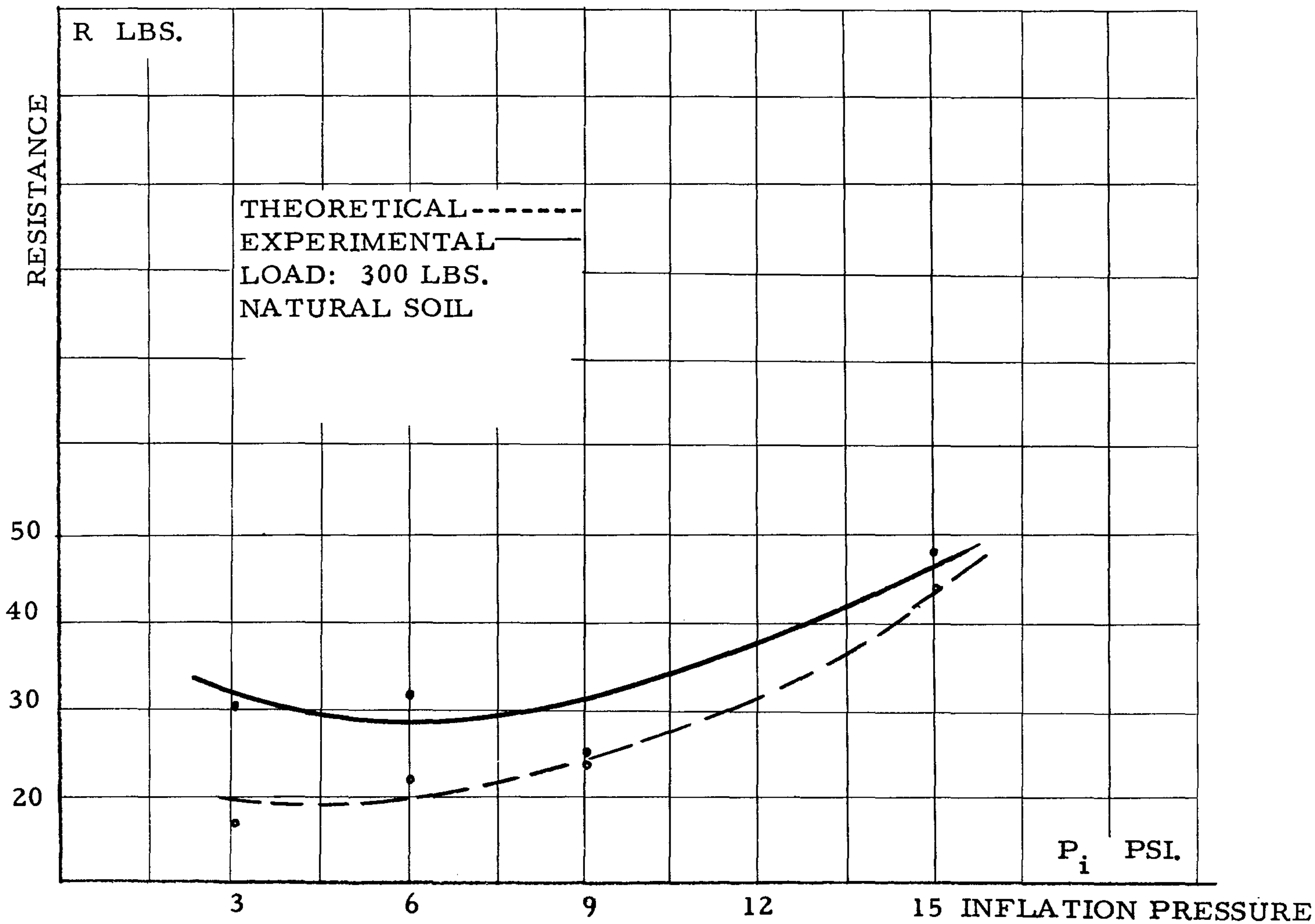

FIGURE 27. EXPERIMENTAL AND THEORETICAL RESULTS, 300-LB. LOAD, NATURAL SOIL

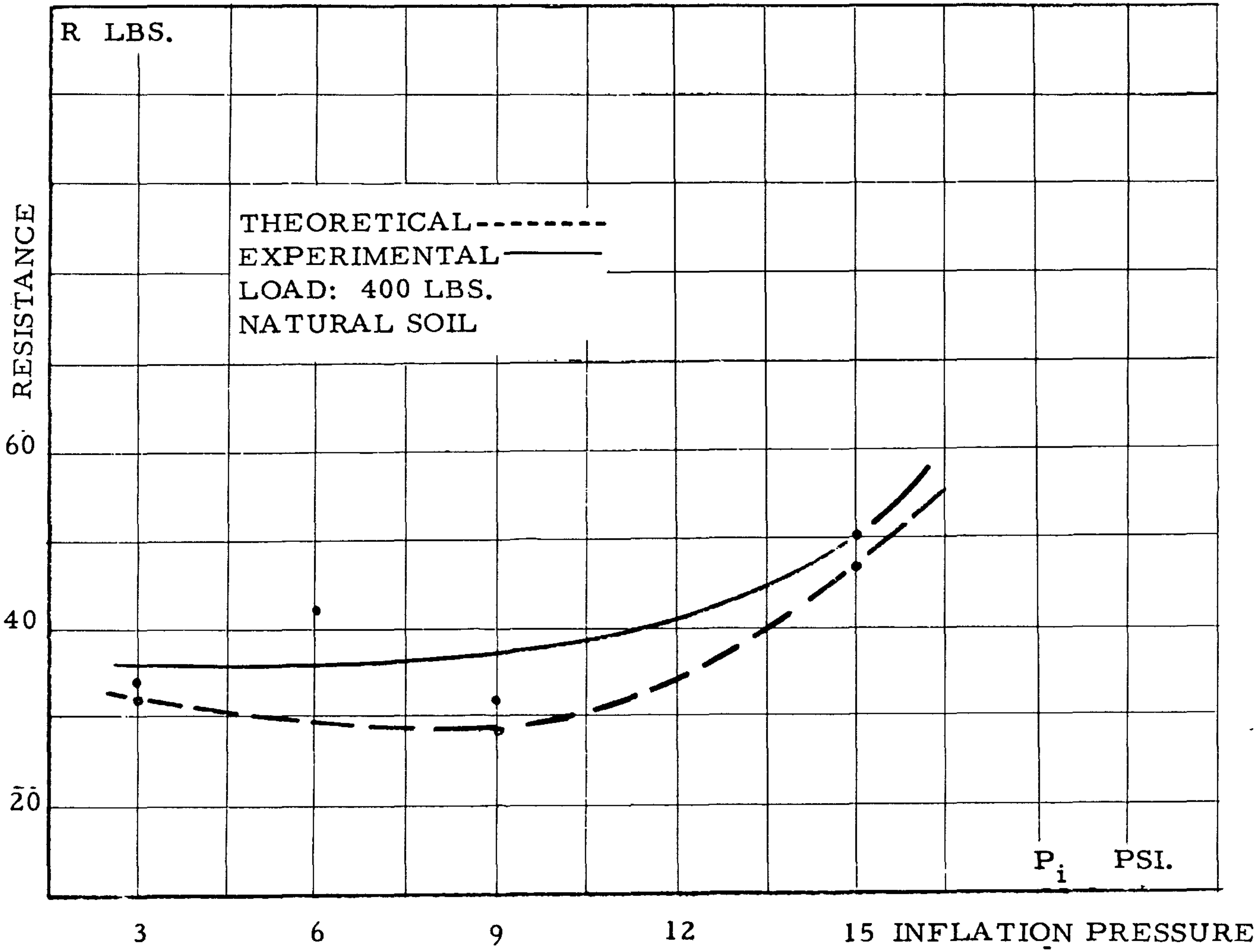

FIGURE 28. EXPERIMENTAL AND THEORETICAL RESULTS, 400-LB. LOAD, NATURAL SOIL

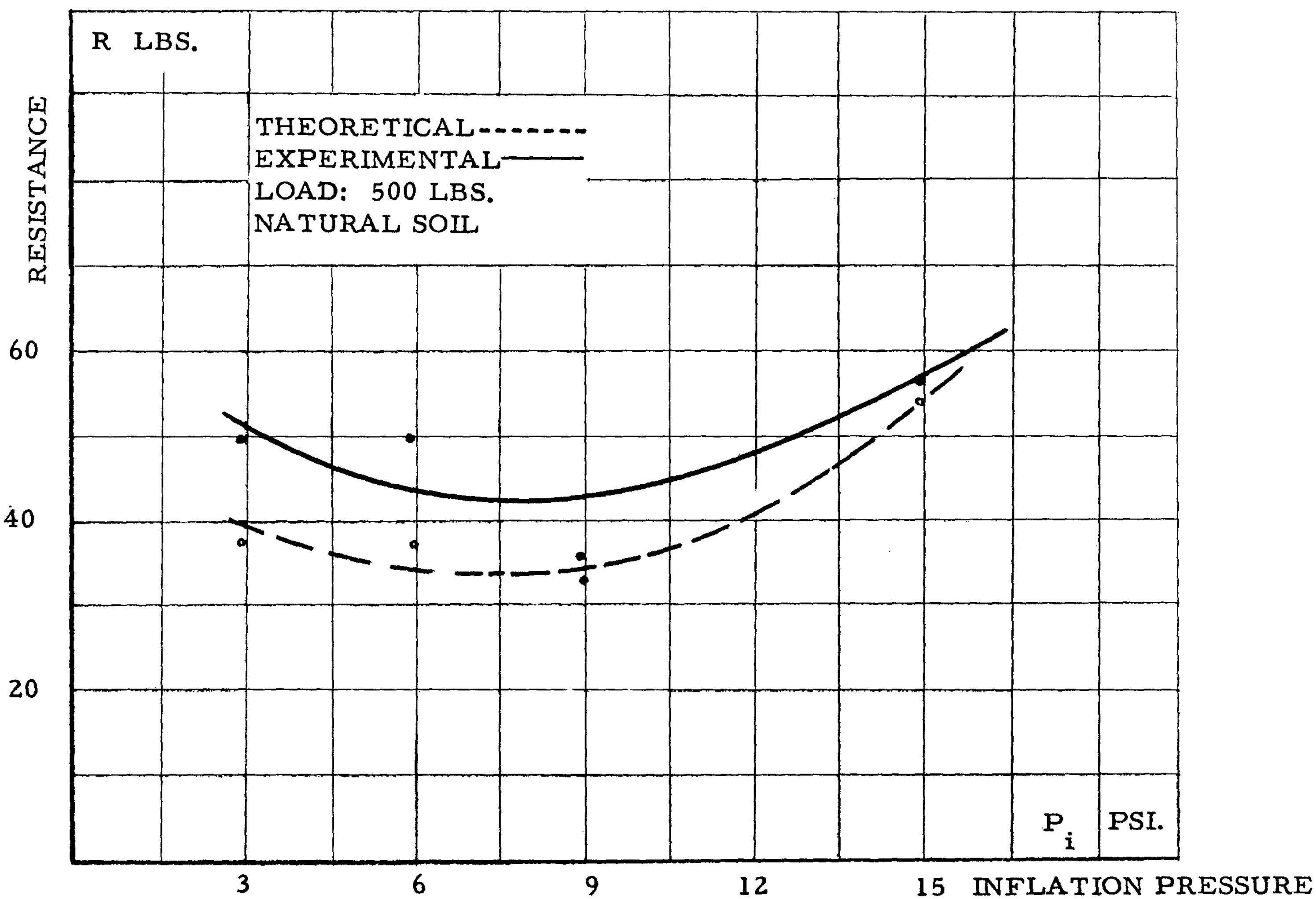

FIGURE 29. EXPERIMENTAL AND THEORETICAL RESULTS, 500-LB. LOAD, NATURAL SOIL

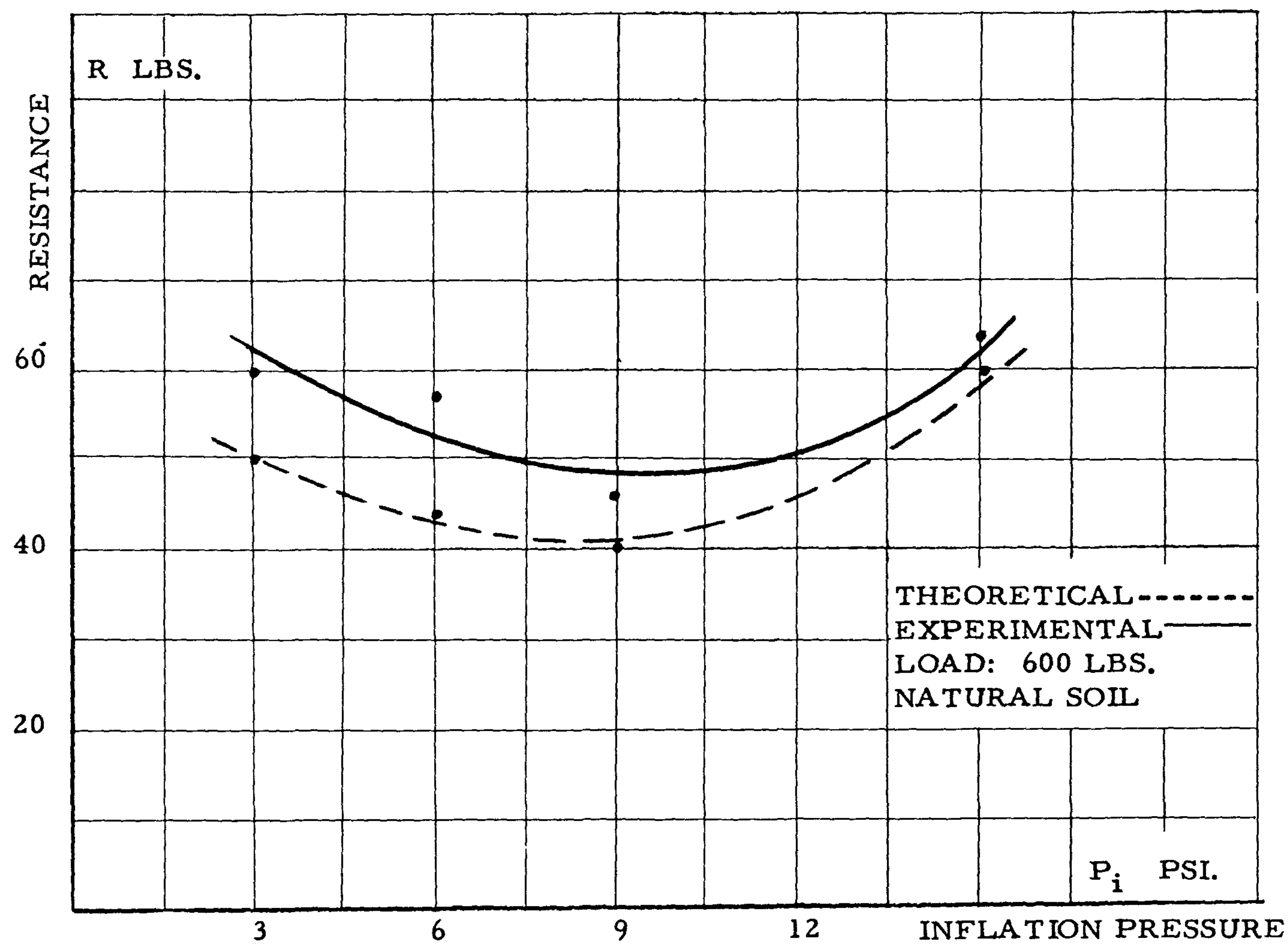

FIGURE 30. EXPERIMENTAL AND THEORETICAL RESULTS, 600-LB. LOAD, NATURAL SOIL

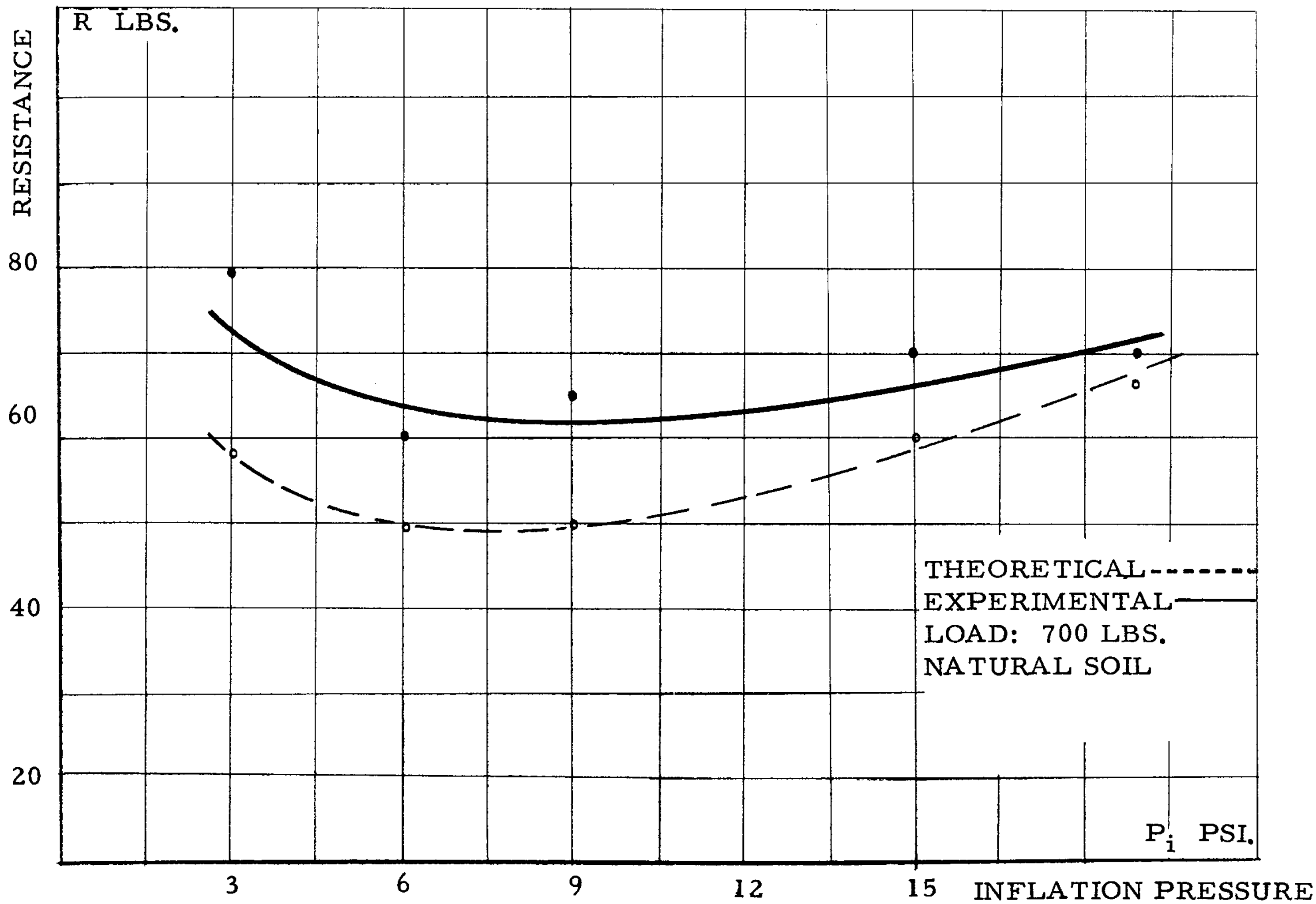

FIGURE 31. EXPERIMENTAL AND THEORETICAL RESULTS, 700-LB. LOAD, NATURAL SOIL

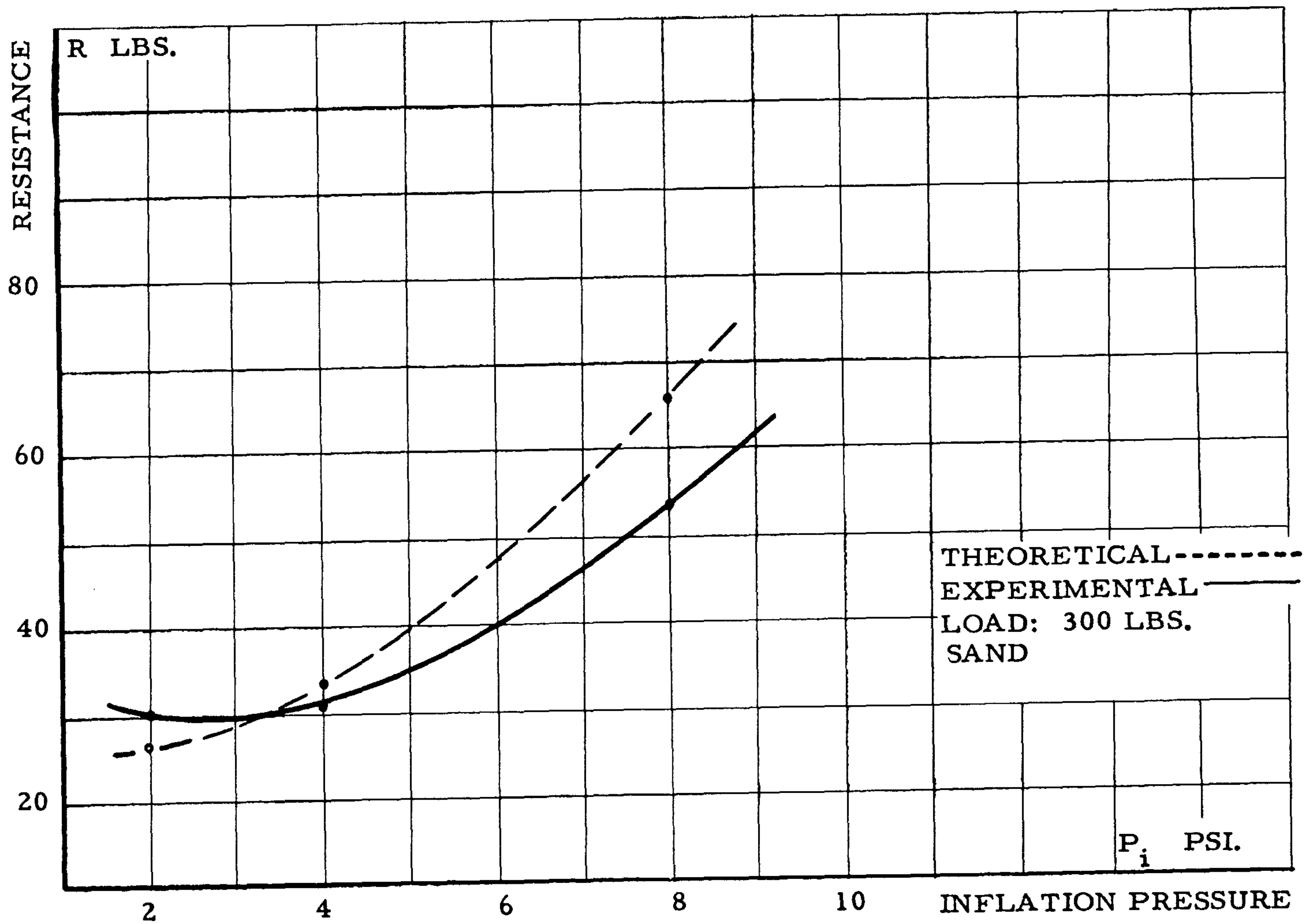

FIGURE 32. EXPERIMENTAL AND THEORETICAL RESULTS, 300-LB. LOAD, SAND

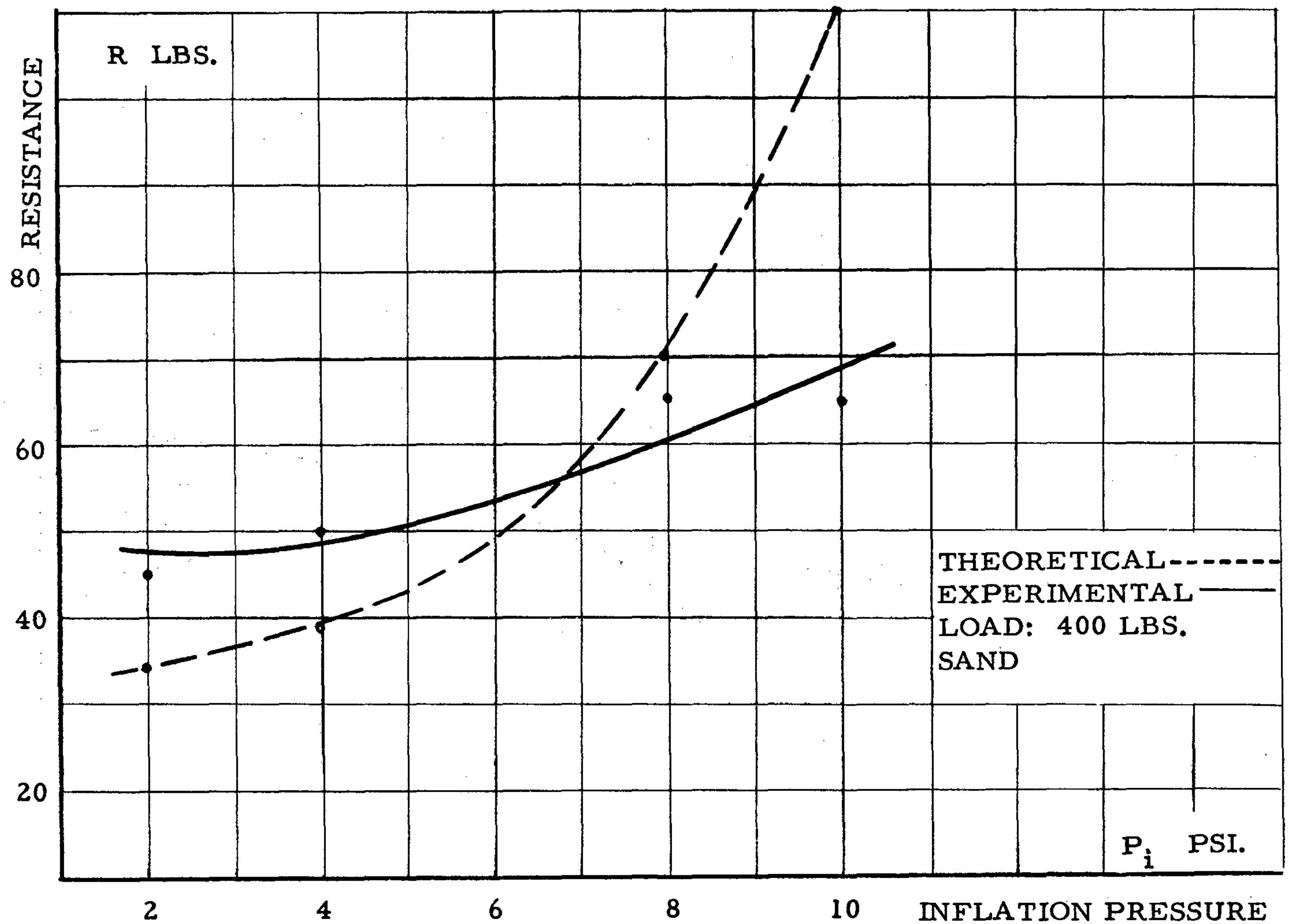

FIGURE 33. EXPERIMENTAL AND THEORETICAL RESULTS, 400-LB. LOAD, SAND

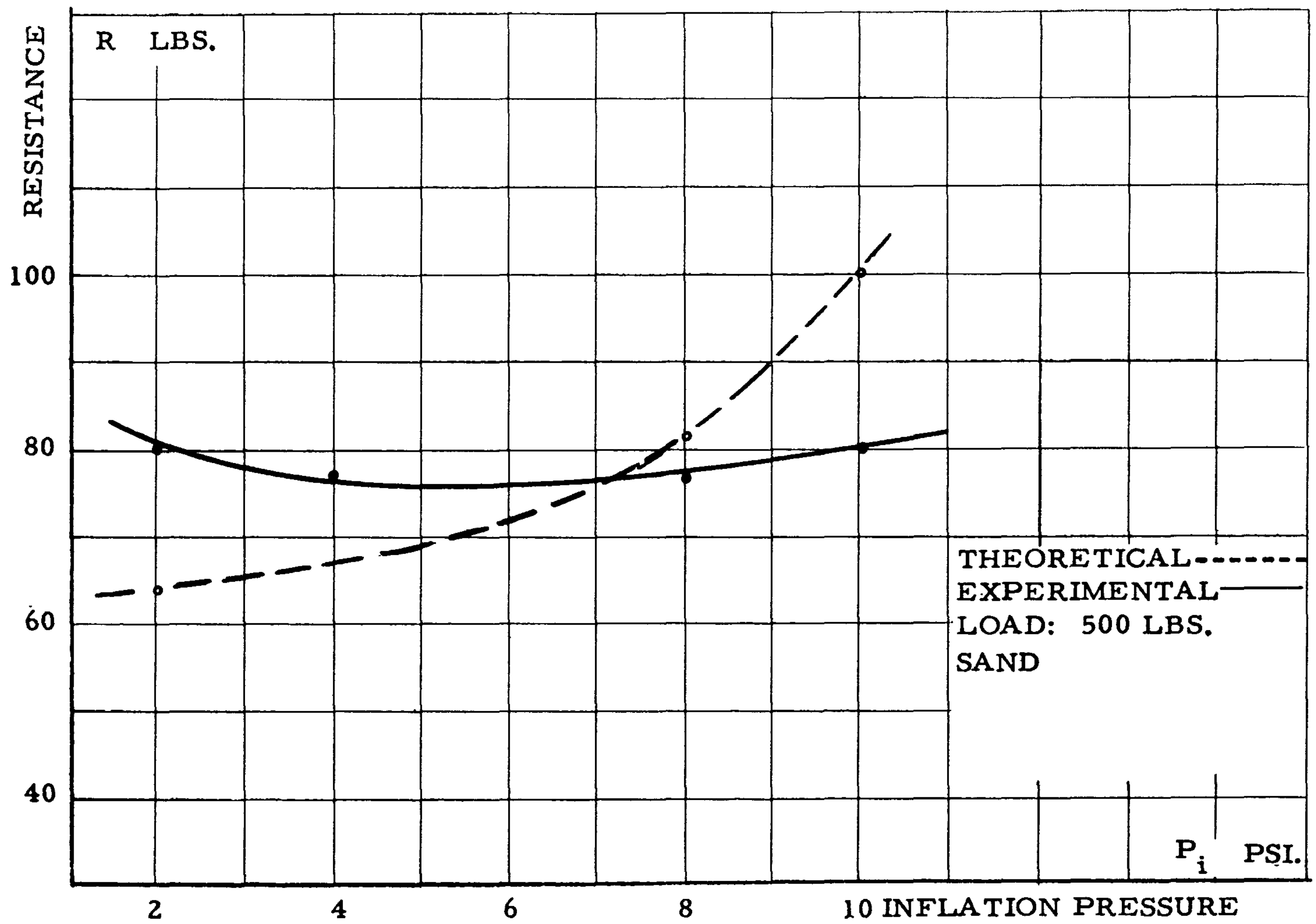

FIGURE 34. EXPERIMENTAL AND THEORETICAL RESULTS, 500-LB. LOAD, SAND

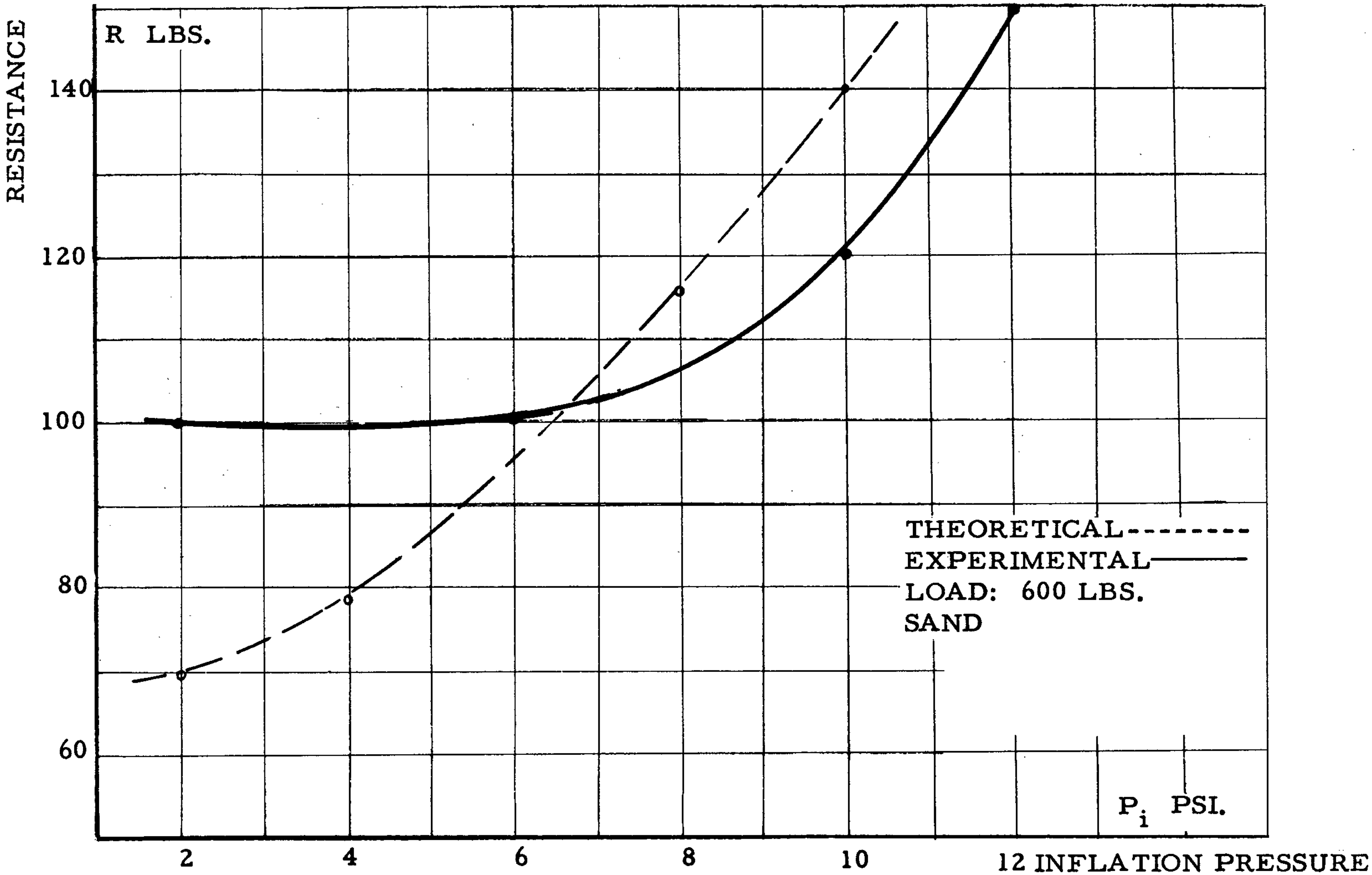

FIGURE 35. EXPERIMENTAL AND THEORETICAL RESULTS, 600-LB. LOAD, SAND

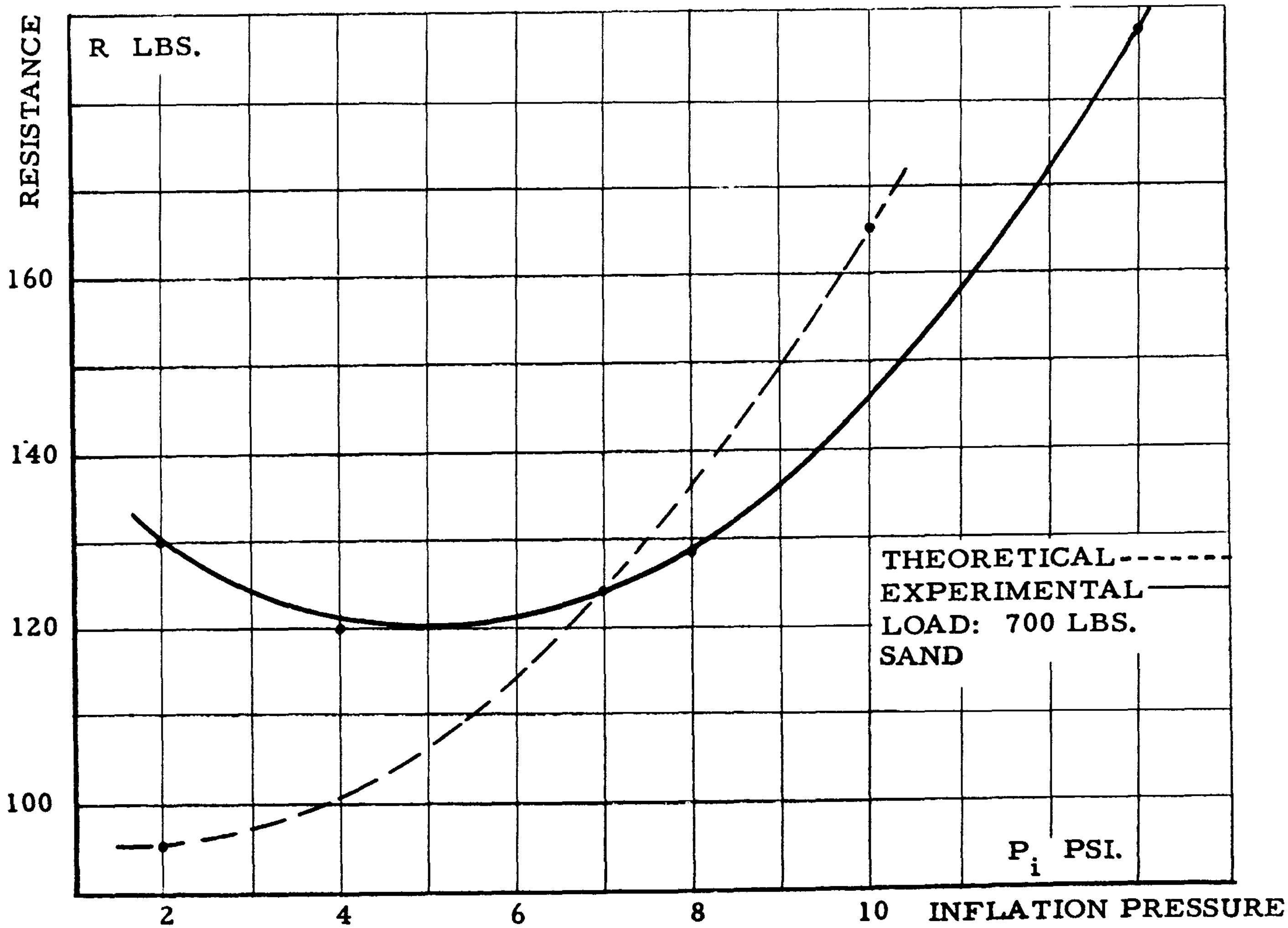

FIGURE 36. EXPERIMENTAL AND THEORETICAL RESULTS, 700-LB. LOAD, SAND

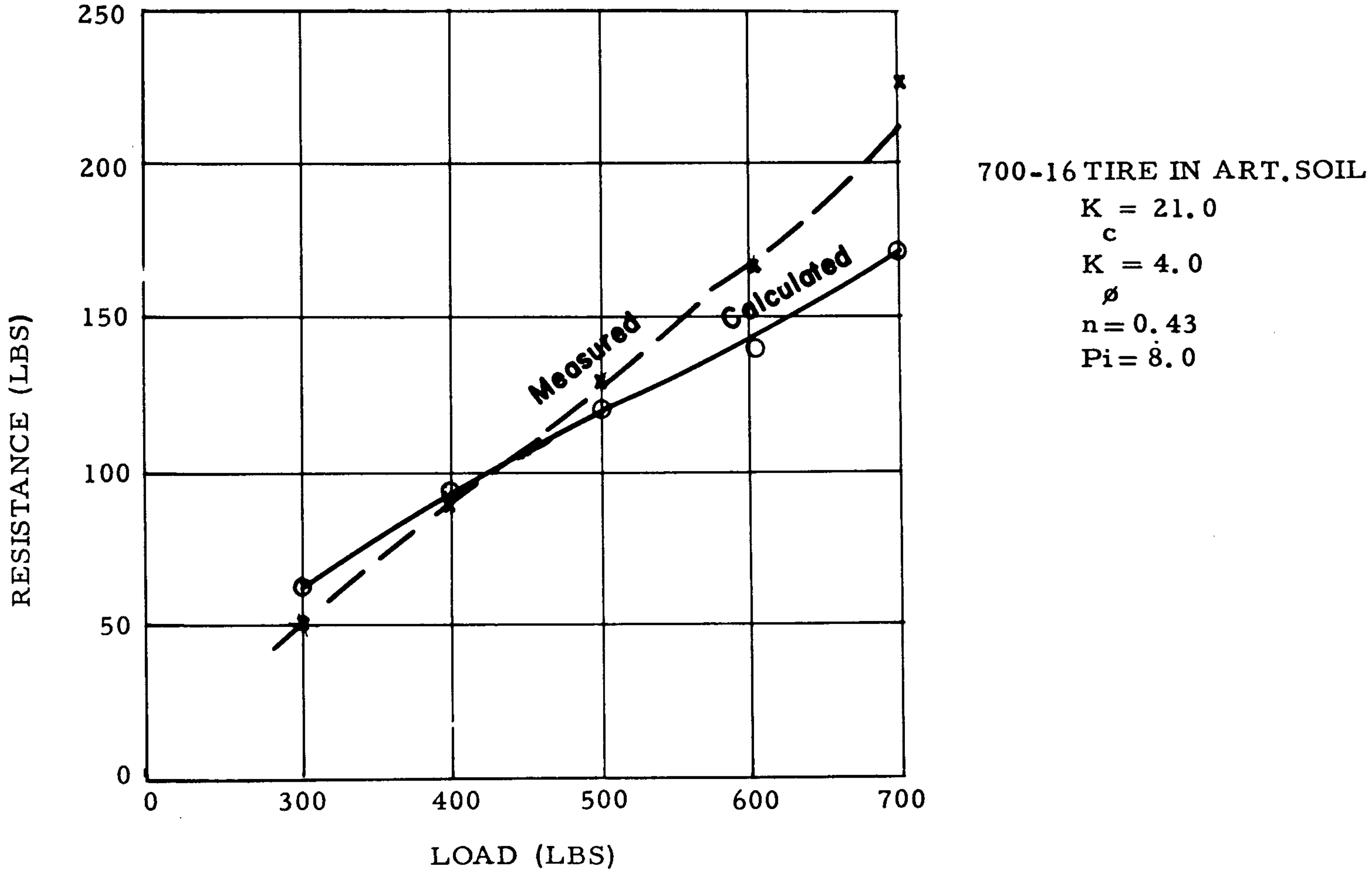

FIGURE 37. EXPERIMENTAL AND THEORETICAL RESULTS, LOAD VS RESISTANCE, ARTIFICIAL SOIL

The second member of equation 9, $R = R_c + R_d$, was handled as follows: The deflection resistance can be expressed:

$$R_d = f_t W$$

where:

$$f_t = \frac{\mu}{(p_i)^a} \quad \text{in equation 8.}$$

or:

$$f_t = \frac{R_d}{W}$$

Figure 38 shows f_t as a function of the inflation pressure. For practical purposes, we may assume that f_t is independent of the weight. The curve, Figure 38, is similar to a hyperbole and can be defined as:

$$f_t = \frac{\mu}{(p_i)^a}$$

The constants, μ and a, characteristics of the test tire, can be obtained by transforming the curve to logarithmic form (Figure 39):

$$\log f_t = \log \mu - a \log p_i$$

In this case, μ and a are taken as 0.10 and 0.64 respectively.

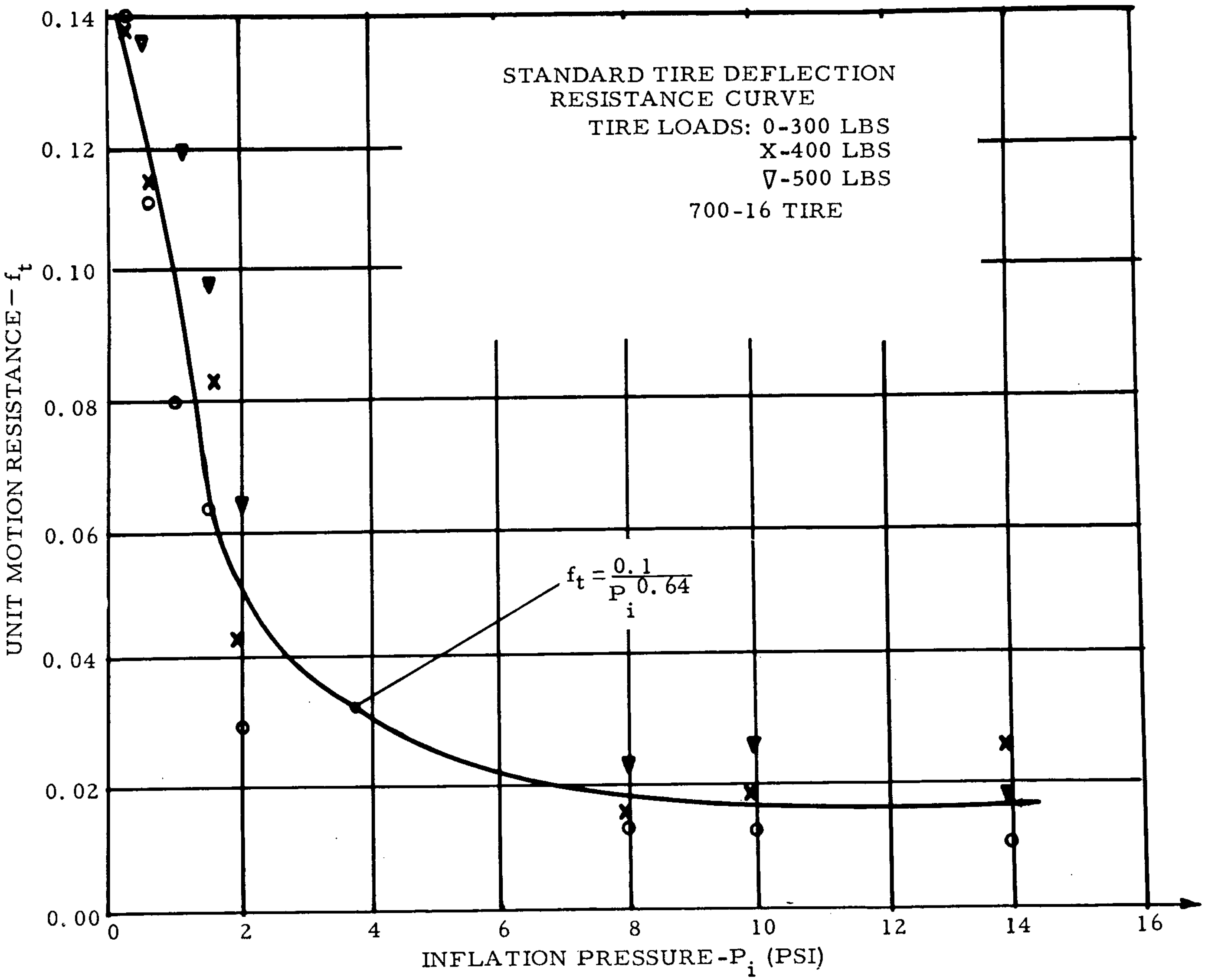

FIGURE 38. STANDARD TIRE DEFLECTION RESISTANCE CURVE

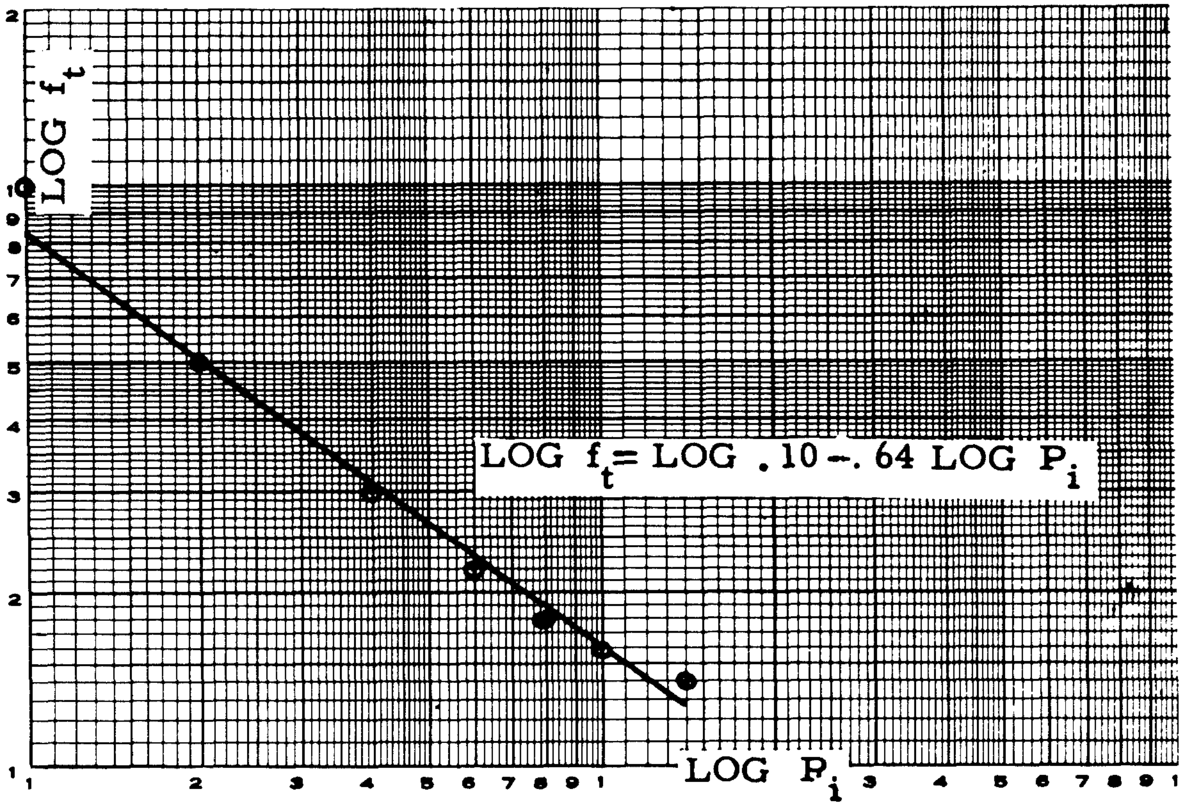

FIGURE 39. LOG P_i VS LOG f_t

Optimum inflation pressure was calculated as described previously, and the results are shown in Figure 40. The values of the theoretical optimum pressure (p_o) are also shown in Figures 27 through 37 and 41 through 45.

A further indication of the accuracy of equation 9 is furnished in Figures 41 through 45. Here, the resistance predicted by equation 9 is plotted but experimental values were used for sinkage instead of theoretical ones. The improved agreement between predicted and experimental resistance as compared to the accuracy obtained in Figures 27 through 37 is due to the elimination of the inaccuracy involved in equation 5.

The critical pressure is plotted for different n and k values, hence for different soils in Figure 46 (k equals $\frac{k_c}{b} + k_\phi$). As expected, the critical pressure increases when the hardness of the soil increases. (The harder the soil, the higher k and n becomes.)

V. CONCLUSIONS:

A. Equation 5 is suitable to predict the sinkage of a tire if the inflation pressure is smaller than the critical pressure lessened by the carcass pressure.

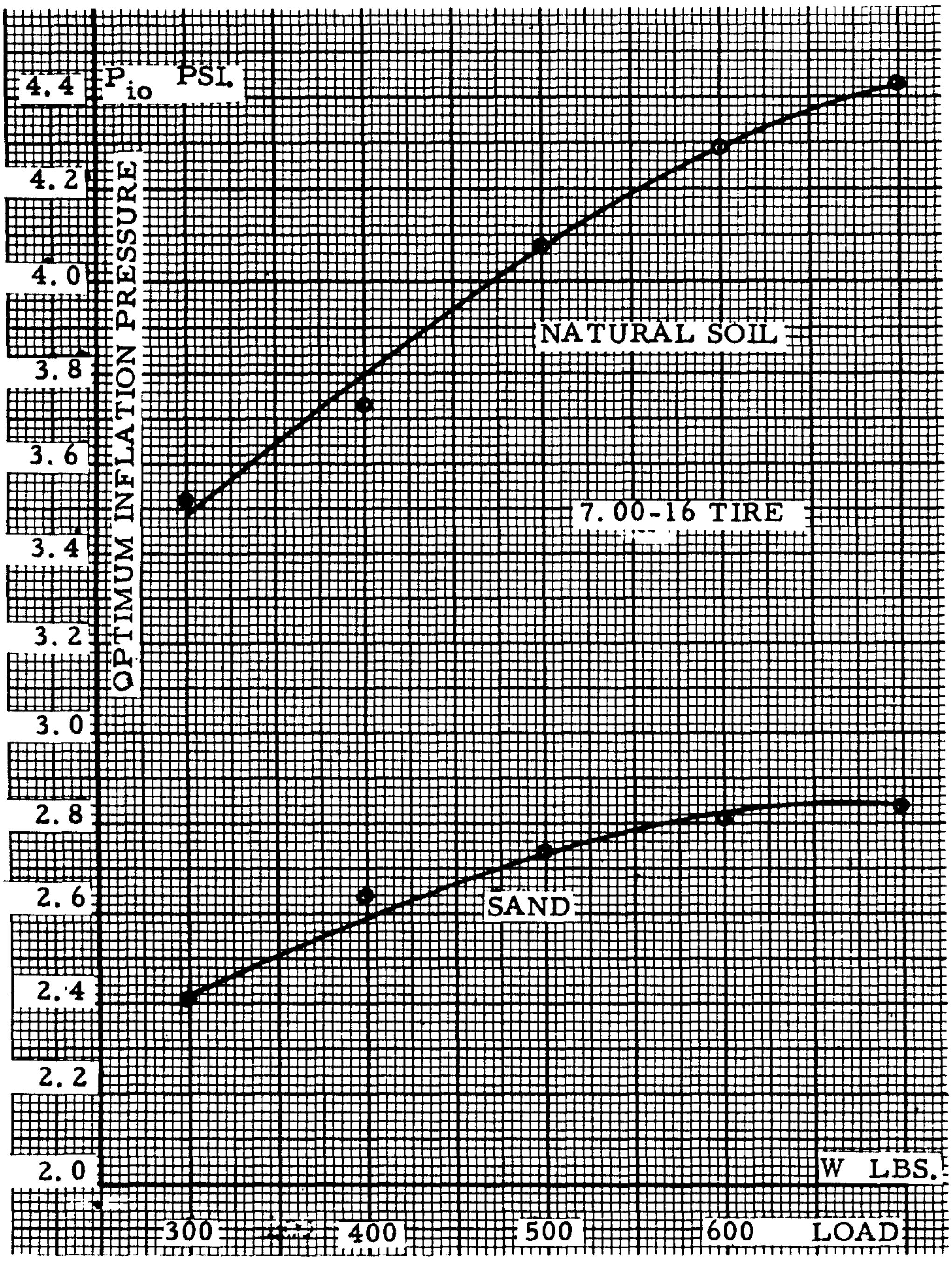

FIGURE 40. OPTIMUM INFLATION PRESSURE VS WEIGHT

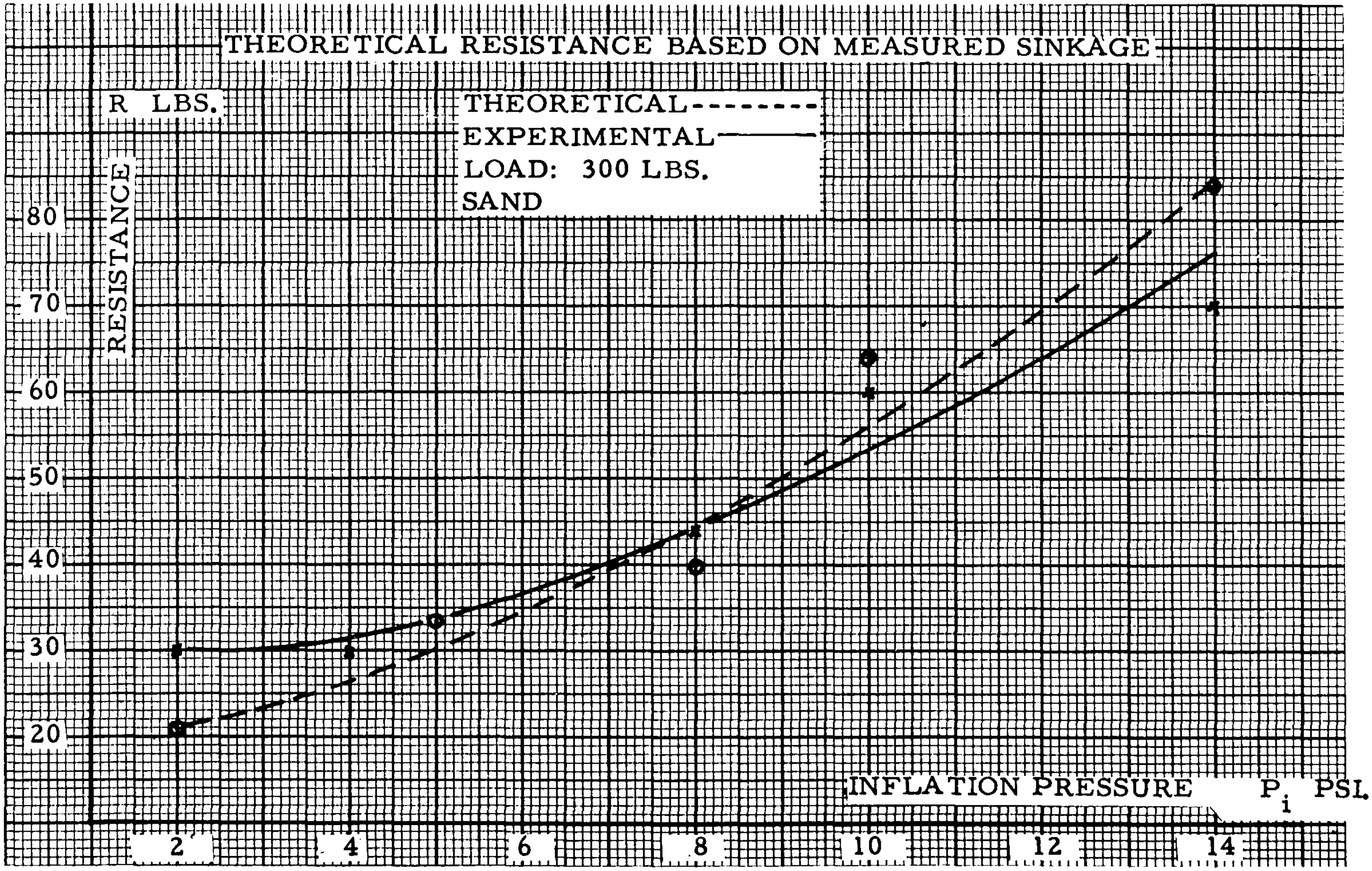

FIGURE 41. EXPERIMENTAL AND THEORETICAL RESULTS, 300-LB. LOAD, SAND (THEORETICAL RESISTANCE BASED ON MEASURED SINKAGE)

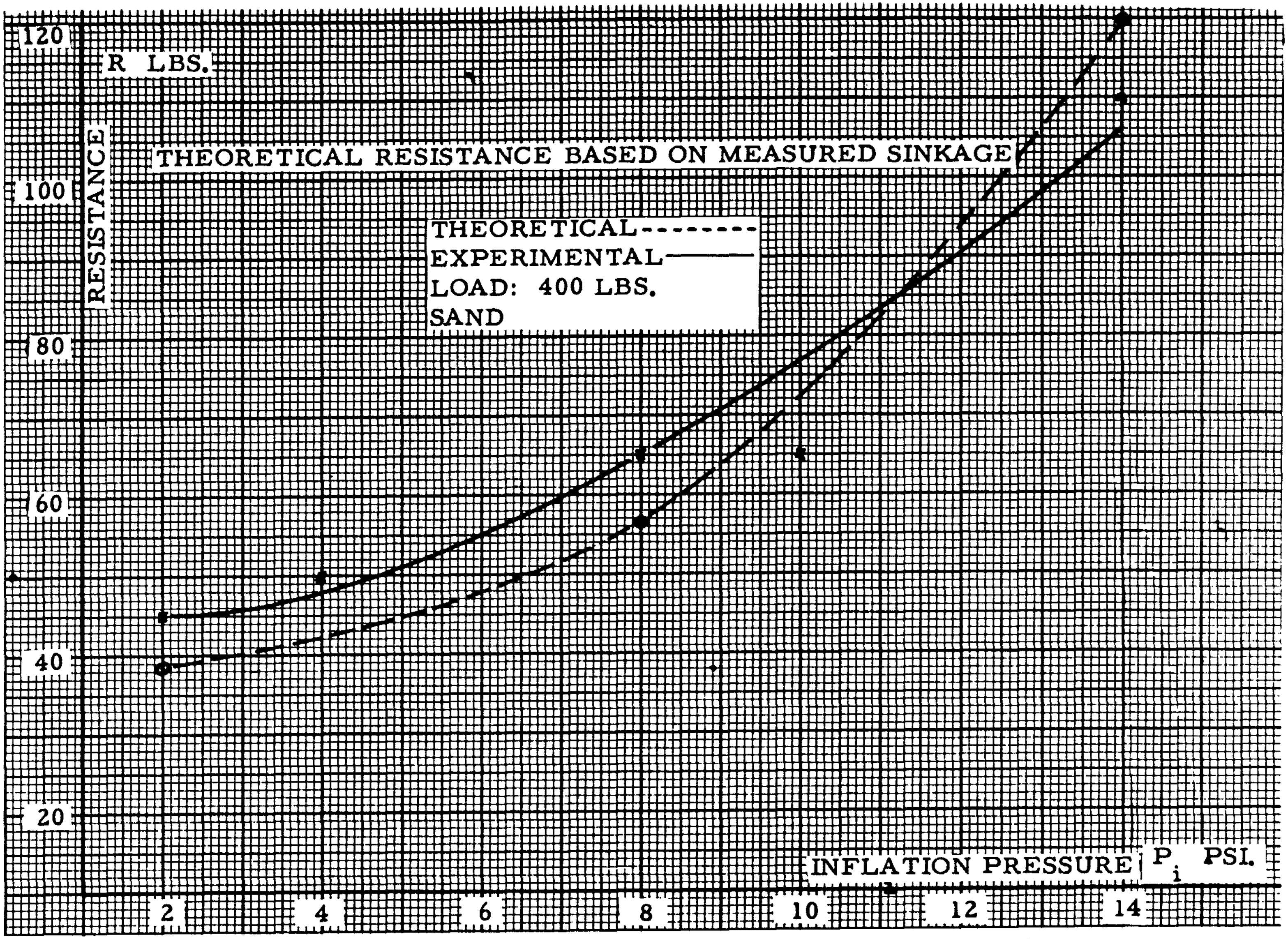

FIGURE 42. EXPERIMENTAL AND THEORETICAL RESULTS, 400-LB. LOAD, SAND (THEORETICAL RESISTANCE BASED ON MEASURED SINKAGE)

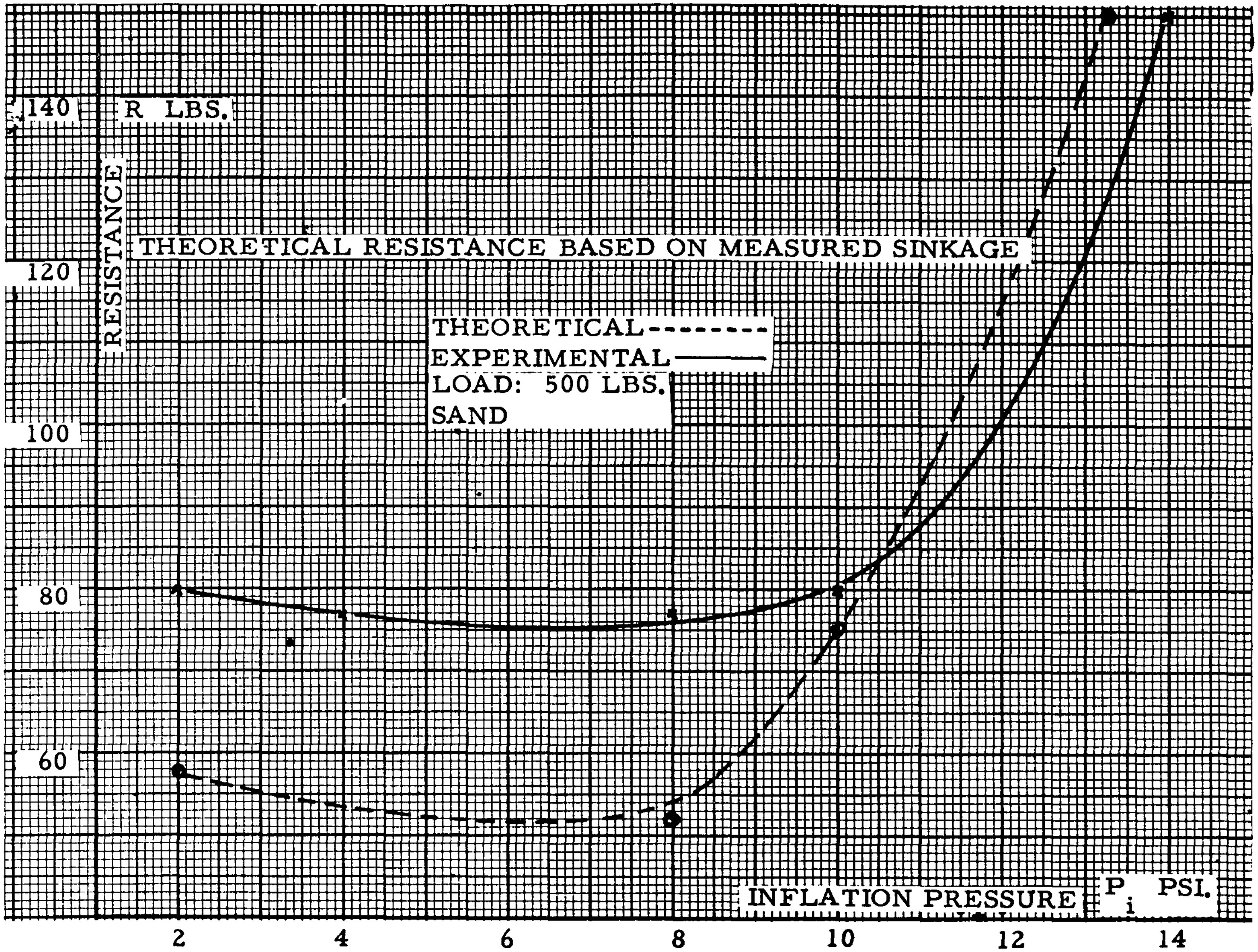

FIGURE 43. EXPERIMENTAL AND THEORETICAL RESULTS, 500-LB. LOAD, SAND (THEORETICAL RESISTANCE BASED ON MEASURED SINKAGE)

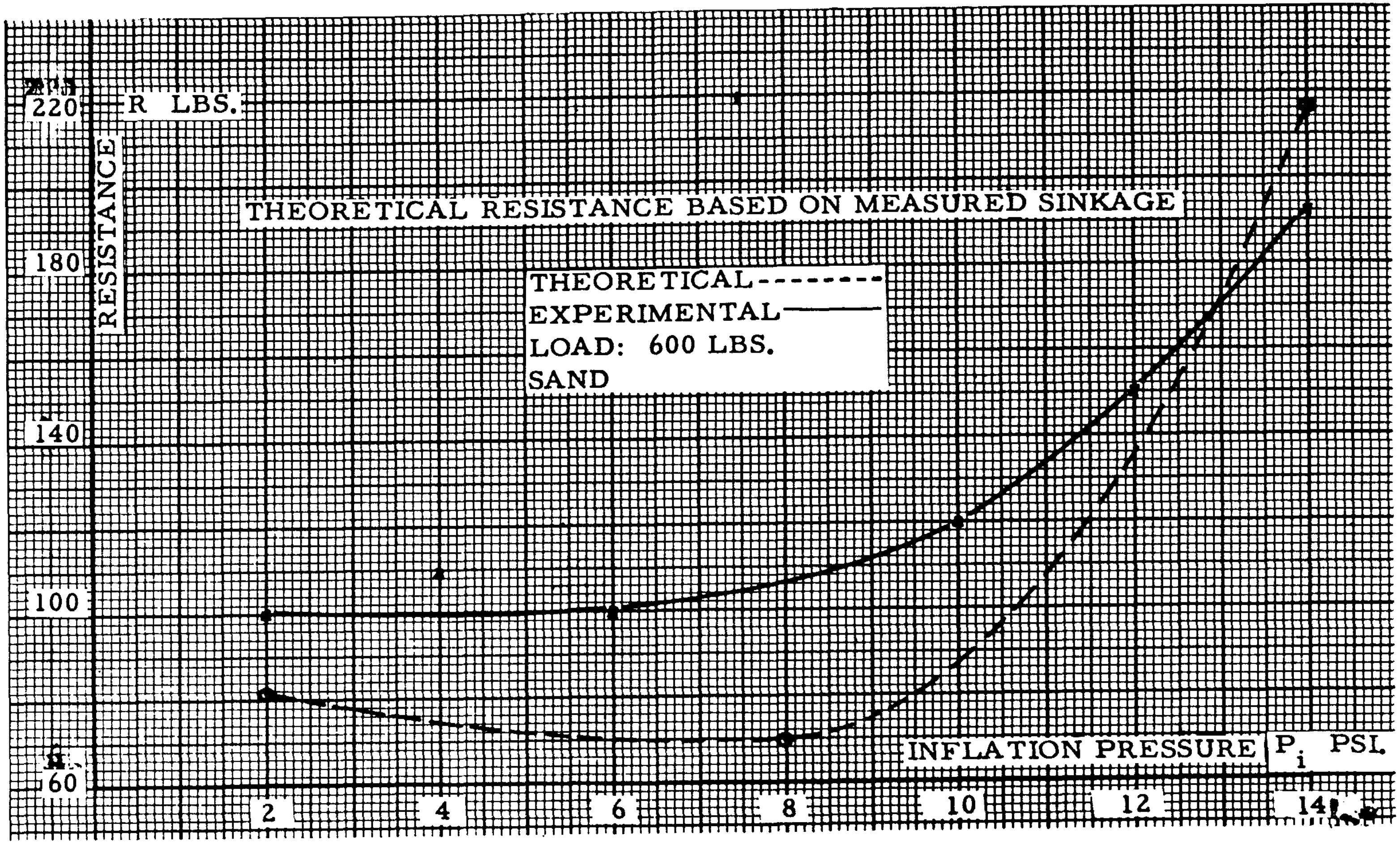

FIGURE 44. EXPERIMENTAL AND THEORETICAL RESULTS, 600-LB. LOAD, SAND (THEORETICAL RESISTANCE BASED ON MEASURED SINKAGE)

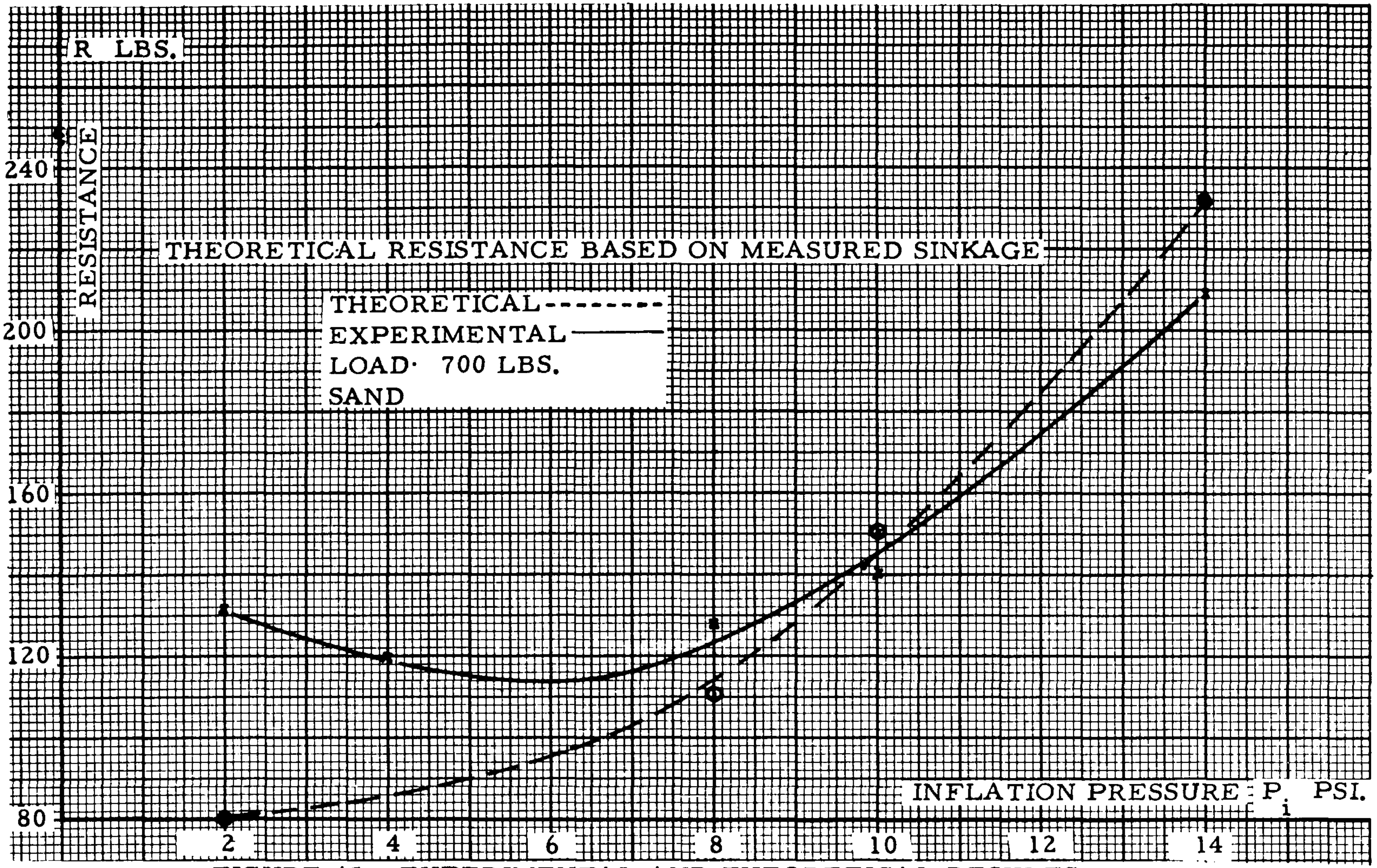

FIGURE 45. EXPERIMENTAL AND THEORETICAL RESULTS, 700-LB. LOAD, SAND (THEORETICAL RESISTANCE BASED ON MEASURED SINKAGE)

CRITICAL INFLATION PRESSURE ABOVE WHICH
THE TIRE BEHAVES LIKE A RIGID WHEEL

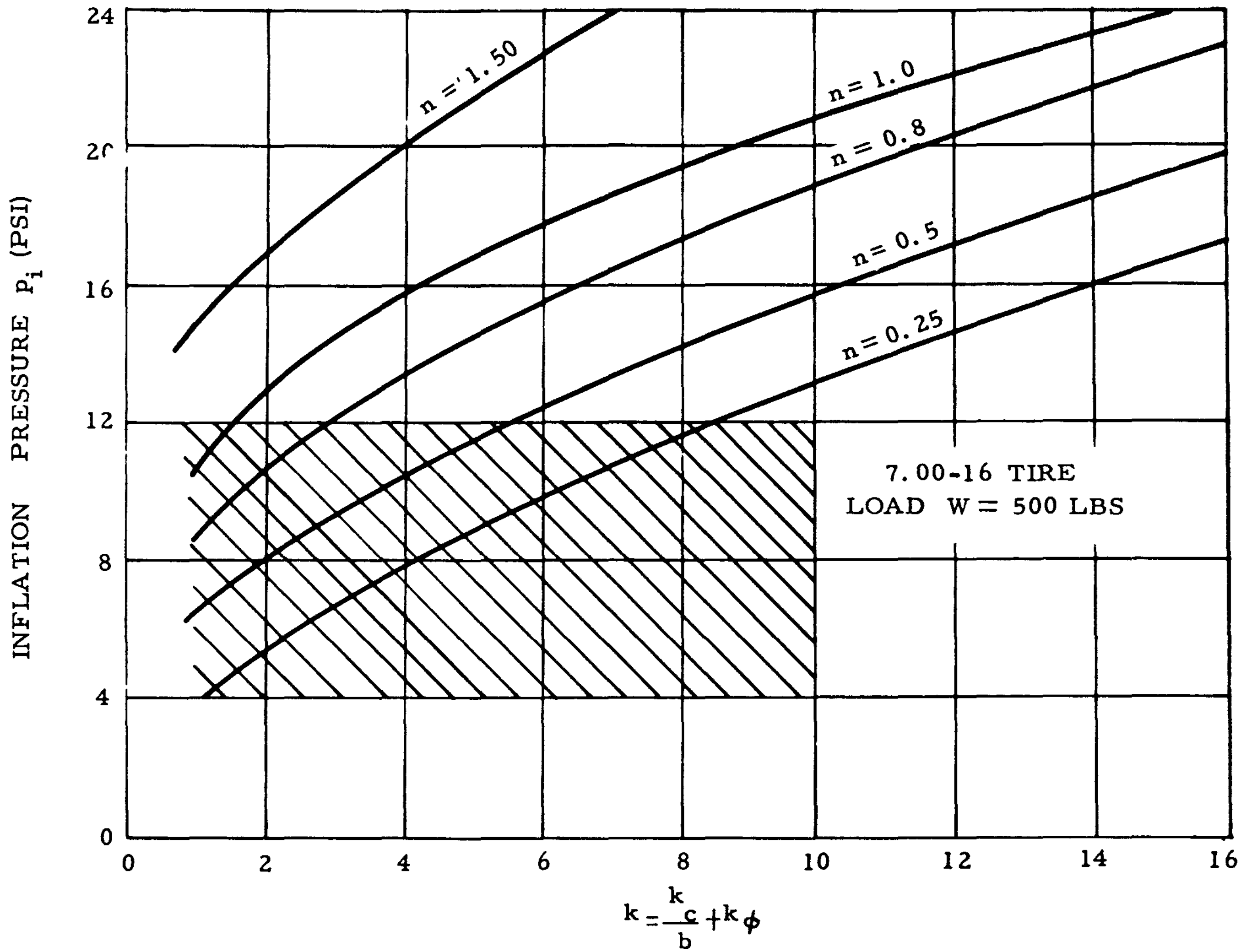

FIGURE 46. CRITICAL PRESSURE FOR DIFFERENT SOILS

Within limits of tire geometry, the sinkage is independent of the load since an increase in load results in an increase of tire deflection rather than an increase in sinkage. This condition persists until the combined inflation and carcass pressure equals or exceeds the critical pressure.

B. Equation 9 can be used for resistance predictions. It shows that a narrow tire is superior to a wide one if the ground contact area is the same. In other words a longitudinally shaped ground contact area is favorable. The increase in weight in order to obtain more traction might not be profitable because of the increase in resistance.

C. There are many pressure reducing devices in use, but the reduction is only useful if the pressure is reduced below the critical pressure. Above the critical pressure, the tire behaves as a rigid wheel and the advantages of less sinkage and more traction obtained from the flat ground contact area are not utilized.

VI. RECOMMENDATIONS:

It is recommended that the procedure outlined in this paper be used for towed pneumatic tire performance prediction.

Further investigations are recommended to establish the importance of the optimum inflation pressure.

The effect of the critical pressure appears to be of utmost importance. It is suggested that it be considered whenever inflation pressure reducing devices are used for improved mobility.

It is also suggested that further investigations be made to establish a scientific method to predict tractive effort or drawbar pull for low inflated tires operating on soft ground.

A series of tests should be conducted to obtain experimental values of μ, a, and p_c for different tires.

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NO.	TITLE
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Superintendent
U. S. Naval Academy
Annapolis, Md 1

Chief of Transportation Corps
Washington 25, D. C. 1

AD________________ ACCESSION NO________________
Ordnance Tank-Automotive Command, Detroit Arsenal, Land Locomotion Laboratory, Center Line, Michigan
ANALYSIS OF TOWED PNEUMATIC TIRES MOVING ON SOFT GROUND (U) - M. G. Bekker and Z. Janosi

Report No. RR-6, March, 1960, 68 pp - Illus
Contract No. DA-20-089-ORD-39246, Proj. No. 5510.11.822
DA Proj. No. 570-05-001 Unclassified Report

A theory by which pneumatic tire sinkage and motion resistance can be calculated has been developed. The formulae based on this theory have been verified by comparing experimental results obtained from tests conducted on several different soils with predicted results using the equations. The agreement between experimental and predicted results was acceptable.

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Ordnance Tank-Automotive Command, Detroit Arsenal, Land Locomotion Laboratory, Center Line, Michigan
ANALYSIS OF TOWED PNEUMATIC TIRES MOVING ON SOFT GROUND (U) - M. G. Bekker and Z. Janosi
LL-62

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